NAME: ______________________________

Instructions: This exam is a 24 hour take home exam. Do not work with anyone else. The solution is due on Friday, May 14 at 9:00 a.m. Print out and hand in any m-files as well as the solution. Also email any m-files to `rawlings@engr.wisc.edu`

Problem 1. Characterize the admissible set. 25 pts.

Recall the set $\mathcal{X}_N$ defined in Chapter 2, which is the set of states in $\mathbb{X}$ for which the MPC regulation problem $\mathbb{P}_N(x)$ has a solution

$$\mathcal{X}_N = \{x \in \mathbb{R}^n \mid \exists \mathbf{u} \in \mathbb{R}^{Nm} \text{ such that } u(k) \in \mathbb{U}, \quad \phi(k; x, \mathbf{u}) \in \mathbb{X}, \quad \forall k \in \mathbb{I}_{0:N-1}, \\ \text{and } \phi(N; x, \mathbf{u}) \in X_f\}$$

For this problem let $\mathbb{X} = \mathbb{R}^n$, and let $\mathbb{U}$ be the hypercube

$$\mathbb{U} = \{u \mid -\mathbf{1} \leq u \leq \mathbf{1}\}$$

in which $\mathbf{1} \in \mathbb{R}^m = \begin{bmatrix} 1 & 1 & \cdots & 1 \end{bmatrix}'$. Characterize the set $\mathcal{X}_N$ for the following cases.

(a) $f(x, u) = Ax + Bu$ is linear, (A, B) is controllable, and $X_f = \{0\}$.

(b) $f(x, u) = Ax + Bu$ is linear, (A, B) is stabilizable, and $X_f = \{0\}$.

Problem 2. i-IOSS and convergence. 25 pts.

Prove that if system

$$x^+ = f(x, w) \qquad y = g(x)$$

is i-IOSS and $w_1(k) \to w_2(k)$ and $y_1(k) \to y_2(k)$ as $k \to \infty$, then

$$x(k; z_1, \mathbf{w}_1) \to x(k; z_2, \mathbf{w}_2) \qquad \text{for all } z_1, z_2$$

In other words, for an i-IOSS system, if the measurements get close and the disturbances are close, the states must eventually get close.

Problem 3. Create your own exercise! 50 pts.

A harried textbook writer asks you to consider the 2-state, 2-input system

$$x^+ = Ax + Bu$$

subject to the constraints

$$x \in \mathbb{X} \subset \mathbb{R}^2, \quad u \in \mathbb{U} \subset \mathbb{R}^2$$

in which you are free to choose A, B, N, Q, R, $\mathbb{U}$, and $\mathbb{X}$. The cost is

$$V_N(x, \mathbf{u}) := \sum_{i=0}^{N-1} \ell(x(i), u(i))$$

in which $\ell(x, u) := (1/2)(|x|_Q^2 + |u|_R^2)$.

(a) Find a set of system and regulator parameters for which unconstrained MPC without terminal cost and without terminal constraint is unstable. Keep $N \geq 3$.

(b) Choose interesting constraint sets and implement constrained MPC without terminal cost and without terminal constraint for the system obtained in the previous part. Is the resultant closed-loop system stable or unstable?

(c) Implement constrained MPC with a terminal equality constraint $x(N) = 0$ for the same problem. Try to characterize $\mathcal{X}_N$ for your system.