ME 155B **HW 3 Solutions** [1] **Fall 2008**

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1. (3.7)

   (a) Plot $x_1(t)$ and $x_2(t)$ vs. t, given $x(0) = [1, 1]^T$, which is shown on the left in Figure 1.

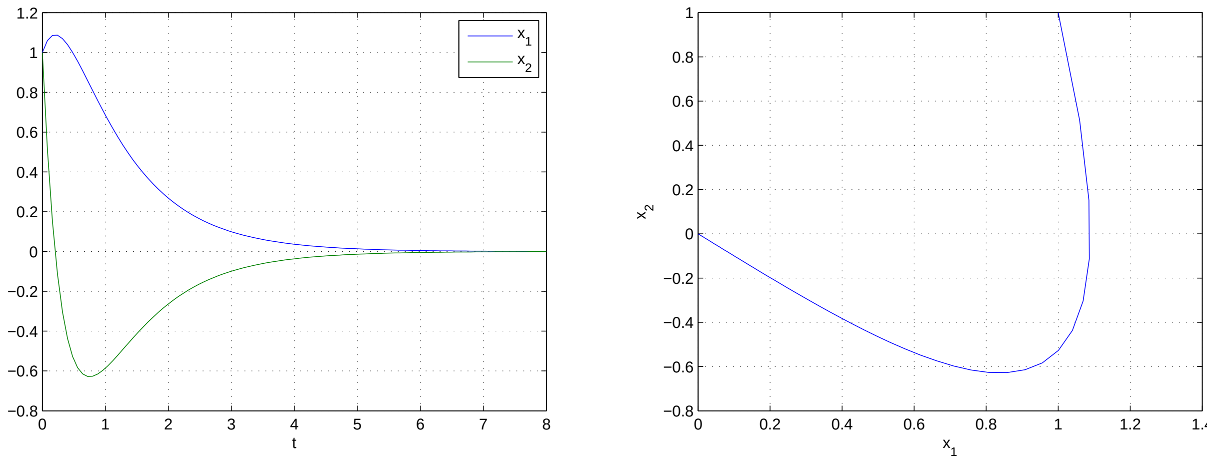


Figure 1: The plot of $x_1(t)$ and $x_2(t)$ vs. t with initial condition $x(0) = [1, 1]^T$ (left), and the plot of $x_2(t)$ vs. $x_1(t)$ for the same $x(0)$ (right).

   (b) Plot $x_2(t)$ vs. $x_1(t)$ for the same $x(0) = [1,\ 1]^T$, which is shown on the right in Figure 1.

   (c) The two eigenvalues are $\lambda_1 = -1$ and $\lambda_2 = -3$, and the corresponding eigenvectors are $v_1 = [0.7071, -0.7071]^T$ and $v_2 = [-0.3162, 0.9487]^T$. The plots are shown in Figure 2 and Figure 3.

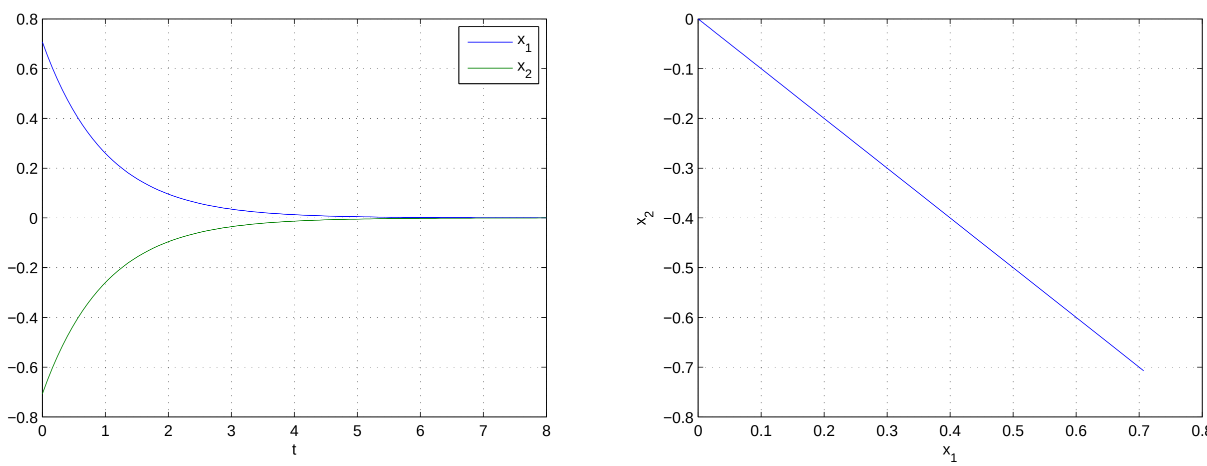
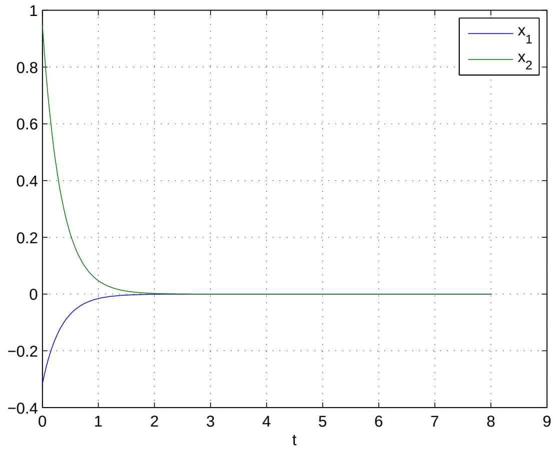


Figure 2: The plot of $x_1(t)$ and $x_2(t)$ vs. t with initial condition $x(0) = [0.7071, -0.7071]^T$ (left), and the plot of $x_2(t)$ vs. $x_1(t)$ for the same $x(0)$ (right).

   (d) This is my `MATLAB` code.

[1] Please email Chunkai at `ckgao@engr.ucsb.edu` if you find any typos in the solutions. Thanks.



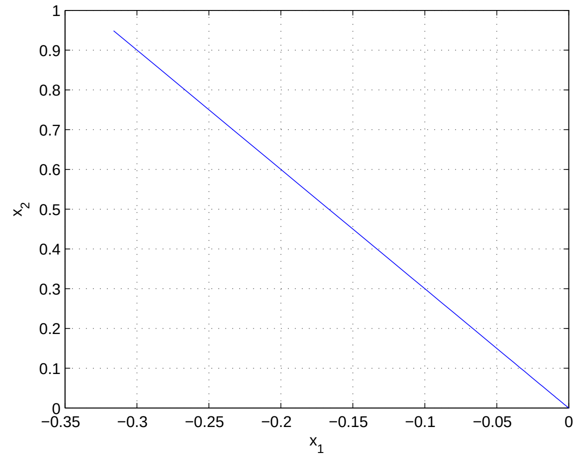


Figure 3: The plot of $x_1(t)$ and $x_2(t)$ vs. t with initial condition $x(0) = [-0.3162, 0.9487]^T$ (left), and the plot of $x_2(t)$ vs. $x_1(t)$ for the same $x(0)$ (right).

```
% Problem 3.7
A=[0 1; -3 -4]; B=[0;0]; C=eye(2); D=[0;0];
mysys=ss(A,B,C,D)

X0=[1;1];
Tfinal=8;
[Y,T,X]=initial(mysys,X0,Tfinal)

% a)
figure;plot(T,X);legend('x_1','x_2');grid on;xlabel('t');

% b)
figure;plot(X(:,1),X(:,2));xlabel('x_1');ylabel('x_2');grid on;

% c)
[V,D] = eig(A)

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
X0=V(:,1); Tfinal=8;
[Y,T,X]=initial(mysys,X0,Tfinal);

% a)
figure;plot(T,X);legend('x_1','x_2');grid on;xlabel('t');

% b)
figure;plot(X(:,1),X(:,2));xlabel('x_1');ylabel('x_2');grid on;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

X0=V(:,2); Tfinal=8;
[Y,T,X]=initial(mysys,X0,Tfinal);

% a)
figure;plot(T,X);legend('x_1','x_2');grid on;xlabel('t');

% b)
figure;plot(X(:,1),X(:,2));xlabel('x_1');ylabel('x_2');grid on;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

2. (3.8)

(a) Plot $x_1(t)$ and $x_2(t)$ vs. t, given $x(0) = [1, 1]^T$, which is shown on the left in Figure 4.

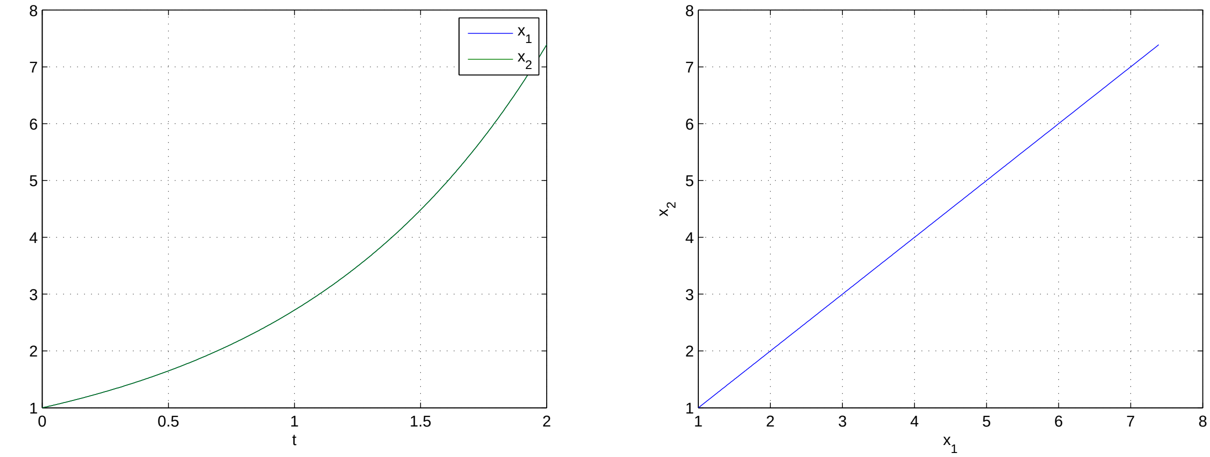


Figure 4: The plot of $x_1(t)$ and $x_2(t)$ vs. t with initial condition $x(0) = [1, 1]^T$ (left), and the plot of $x_2(t)$ vs. $x_1(t)$ for the same $x(0)$ (right).

(b) Plot $x_2(t)$ vs. $x_1(t)$ for the same $x(0) = [1,\ 1]^T$, which is shown on the right in Figure 4.

(c) The two eigenvalues are $\lambda_1 = -1$ and $\lambda_2 = 1$, and the corresponding eigenvectors are $v_1 = [-0.7071, 0.7071]^T$ and $v_2 = [0.7071, 0.7071]^T$. The plots are shown in Figure 5 and Figure 6.

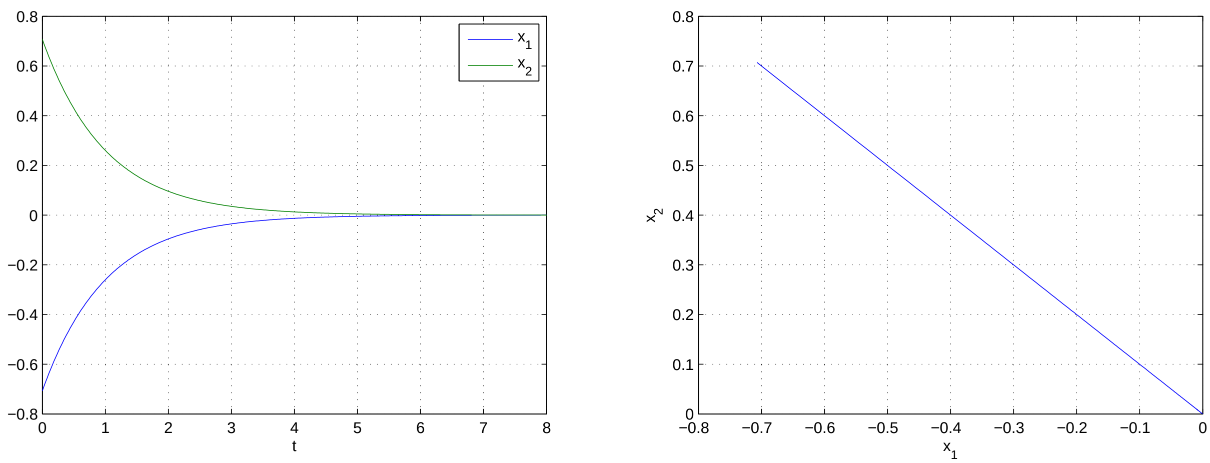
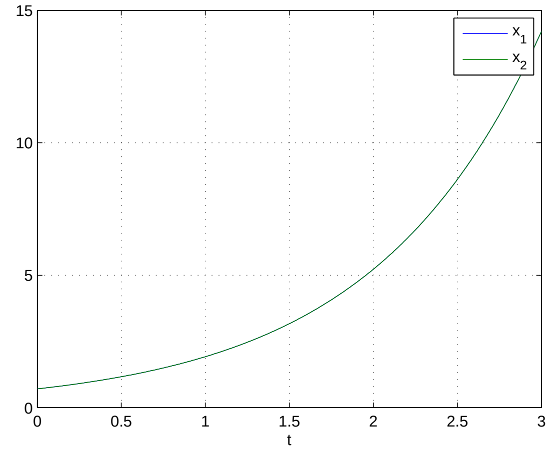


Figure 5: The plot of $x_1(t)$ and $x_2(t)$ vs. t with initial condition $x(0) = [-0.7071, 0.7071]^T$ (left), and the plot of $x_2(t)$ vs. $x_1(t)$ for the same $x(0)$ (right).



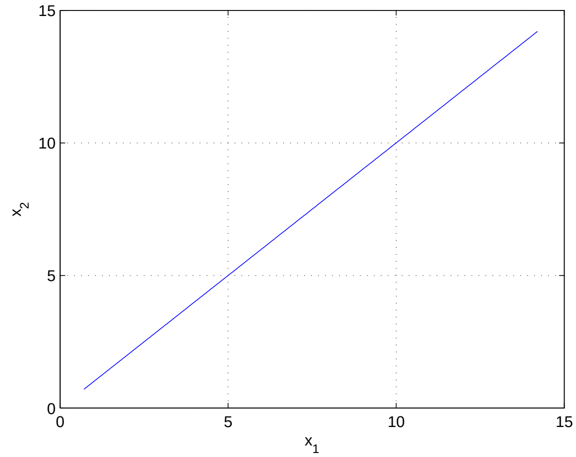


Figure 6: The plot of $x_1(t)$ and $x_2(t)$ vs. t with initial condition $x(0) = [0.7071, 0.7071]^T$ (left), and the plot of $x_2(t)$ vs. $x_1(t)$ for the same $x(0)$ (right).

3. (3.14) Servo with flexible shaft

   Below is the `MATLAB` code we used in `Homework 1` to calculate the state space model.

```
% Define constants
Km = 0.05; R = 1.2; L = 0.05; Jm = 8e-4; J = 0.02;
N = 12; K = 500;

% Define A, B, C matrices
A = [0 0 1 0 0;
    0 0 0 1 0;
    -K*((1/J)+(1/(N^2*Jm))) 0 0 0 (Km/(Jm*N));
    K/J  0 0 0 0;
    0 0 -Km*N/L  -Km*N/L  -R/L];

B = [0; 0; 0; 0; 1/L];

C = eye(5); C(1,2) = 1; C(3,4) = 1;
D = zeros(5,1);
SS = ss(A,B,C,D);
```

   Where the states are $(\Delta, \theta_2, \Omega, \omega_2, i)'$. To calculate the transfer function of θ_1/v and θ_2/v, we need set θ_1 and θ_2 as the output of our system. So we continue with the following `MATLAB` codes:

```
newC=C(1:2,:);
newD=D(1:2,:);
[num,den]=ss2tf(A,B,newC,newD,1);

% tf theta_1
tf(num(1,:),den)

%tf theta_2
tf(num(2,:),den)
```

   We get by `MATLAB`

```
Transfer function:
1.421e-014 s^4 - 1.819e-011 s^3 + 104.2 s^2 + 1.397e-009 s + 2.604e006
----------------------------------------------------------------------
      s^5 + 24 s^4 + 2.94e004 s^3 + 7.042e005 s^2 + 1.562e006 s


Transfer function:
-3.553e-015 s^4 + 3.638e-012 s^3 + 1.63e-009 s + 2.604e006
------------------------------------------------------------
s^5 + 24 s^4 + 2.94e004 s^3 + 7.042e005 s^2 + 1.562e006 s
```

which says essentially

$$\begin{aligned} \mathrm{tf}(\theta_1; v) &= \frac{104.2s^2 + 2.604 * 10^6}{s^5 + 24s^4 + 2.94 * 10^4 s^3 + 7.042 * 10^5 s^2 + 1.562 * 10^6 s}, \\ \mathrm{tf}(\theta_2; v) &= \frac{2.604e006}{s^5 + 24s^4 + 2.94e004s^3 + 7.042e005s^2 + 1.562e006s}. \end{aligned}$$

4. (3.23)

(a) Calculate e^{At}

$$\begin{aligned} (sI - A)^{-1} &= \begin{bmatrix} s & -1 \\ -1 & s \end{bmatrix}^{-1} = \begin{bmatrix} \frac{1}{2}(\frac{1}{s-1} + \frac{1}{s+1}) & \frac{1}{2}(\frac{1}{s-1} - \frac{1}{s+1}) \\ \frac{1}{2}(\frac{1}{s-1} - \frac{1}{s+1}) & \frac{1}{2}(\frac{1}{s-1} + \frac{1}{s+1}) \end{bmatrix} \\ e^{At} &= \begin{bmatrix} \frac{e^t}{2} + \frac{e^{-t}}{2} & \frac{e^t}{2} - \frac{e^{-t}}{2} \\ \frac{e^t}{2} - \frac{e^{-t}}{2} & \frac{e^t}{2} + \frac{e^{-t}}{2} \end{bmatrix} \\ Ce^{At} &= \begin{bmatrix} e^{-t} & -e^{-t} \end{bmatrix} \end{aligned}$$

Let $x^* = [1, 1]^T$, then $Ce^{At}x^* = 0$, for all $t \geq 0$.

(b)

$$\begin{aligned} \mathbb{O} &= \begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \\ \mathbb{O}x^* &= \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \end{aligned}$$

(c)

$$\begin{aligned} Ax^* &= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} = x^*, \\ Cx^* &= \begin{bmatrix} 1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = 0. \end{aligned}$$