ME155B , F08, HW 6 addendum

Problem 1.

For the input $u^* = 1$, a) find the associated equilibrium state x^* in the system below. b) linearize the system at this equilibrium.

$$\dot{x} = xu - 1$$
$$y = xu^2$$

Problem 2.

Consider the system $\dot{x} = f(x,u);\ y = h(x,u); x(0) = 0$. For the input $u = 1$ the output is $y(t) = e^{-t}$. For the input $u = -1$ the output is $y(t) = -e^{-t}$. Is the system linear? Explain.

Problem 3.

In the frequency domain, let Y(s) = U(s)V(s). What is y(t) in terms of u(t) and v(t) (write the relationship as an integral expression, and be precise about the limits of the integral)?

Problem 4.

For the system [A,B,C,D] (that is a state space realization as usual defined by the matrices A,B,C,D), derive the relationship $y(t) = Ce^{At}x(0) + \int_0^t Ce^{A(t-\tau)}Bu(\tau)d\tau + Du(t)$

(do not quote the results in the book, but show this from first principles, i.e. that the expression above does indeed satisfy the system equations)

Problem 6.

The pair $A = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix}$ $C = \begin{bmatrix} 1 & -1 \end{bmatrix}$ is unobservable. Find the unobservable mode (i.e. find the initial state direction that produces a zero output)

Problem 5.

Refer to the system and solution formula of problem 4. Assume for simplicity that D=0. We want to devise an algorithm that solves for the unknown initial condition x(0) given the input and output functions over an interval of time, i.e. given u(t), y(t) over 0<= t <=T. You will need to assume that the system is observable, and this assumption will have to be used in your calculations.

Note: Careful: you cannot invert C! typically, y is a vector of a smaller dimension than x. Hint: Once you set up an equation that needs to be solved for x(0), try taking time derivatives of that equation.