1. (7.37) We first use `MATLAB` to calculate the observer gain,

```
A=[0 1; -1 -1]; B=[0;1]; C=[1 1];
obs_pole=[-3+sqrt(3)*i -3-sqrt(3)*i]
K = place(A',C',obs_pole)
G=K'
```

where we get $G = \begin{pmatrix} -6 & 11 \end{pmatrix}^T$. So the second-order observer is

$$\dot{\hat{x}} = A\hat{x} + Bu + G(y - C\hat{x}),$$

where A, B, C and G are all known.
Putting the original system together with the observer, we get a new system

$$\frac{d}{dt}\begin{pmatrix} x \\ \hat{x} \end{pmatrix} = \begin{pmatrix} A & 0 \\ GC & A - GC \end{pmatrix}\begin{pmatrix} x \\ \hat{x} \end{pmatrix} + \begin{pmatrix} B \\ B \end{pmatrix} u$$

Simulate this system with given initial conditions and input, the response is shown in Figure 1.

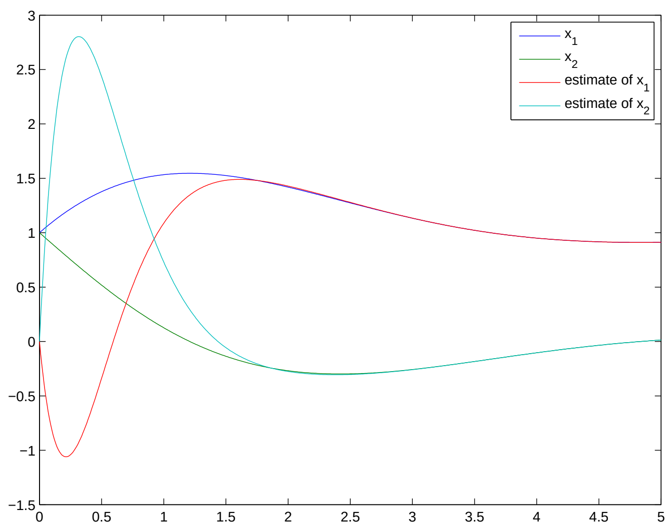


Figure 1: $x(t)$ and $\hat{x}(t)$ with given initial conditions and input.

[1] Please email Chunkai at `ckgao@engr.ucsb.edu` if you find any typos in the solutions. Thanks.

2. (7.46)

(a) Design an optimal observer, using θ_2, ω_2 and i as measurement.

```
% Define constants
Km = 0.05; R = 1.2; L = 0.05; Jm = 8e-4; J = 0.02; N = 12; K = 500;
% Define A, B, C matrices
A = [0 0 1 0 0;
     0 0 0 1 0;
     -K*((1/J)+(1/(N^2*Jm))) 0 0 0 (Km/(Jm*N));
     K/J  0 0 0 0;
     0 0 -Km*N/L  -Km*N/L  -R/L];
B = [0; 0; 0; 0; 1/L];
C = eye(5); C(1,2) = 1; C(3,4) = 1;
D = zeros(5,1);

%%%% (a) %%%%
Ca=[0 1  0 0 0
    0 0  0 1 0
    0 0  0 0 1]
V=diag([0.001^2, 0.05^2, 0.005^2])
w0=5;
w=[0 0 0 w0 0]';
W=w*w'
[P,L,Gt] = care(A',Ca',W,V)
G=Gt'
```

Run MATLAB, we then get

$$G = \begin{pmatrix} -0.0349 & -0.0001 & 0.007 \\ 23.0328 & 0.444 & -6.9184 \\ -1.8526 & -0.2216 & 0.8275 \\ 1109.9584 & 82.8735 & -513.1851 \\ -172.9599 & -5.1319 & 71.0915 \end{pmatrix}$$

So the optimal observer is

$$\dot{\hat{x}} = A\hat{x} + Bu + G(y - Ca\hat{x}),$$

where A, B, Ca and G are all given in the MATLAB codes, and can be easily checked when running MATLAB.

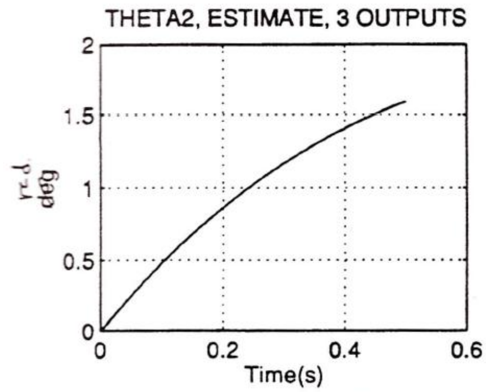
b) For $w(t) = 5\,u_0(t)$, the state at $t = 0+$ is just $[0\ 0\ 0\ 5\ 0]^T$.

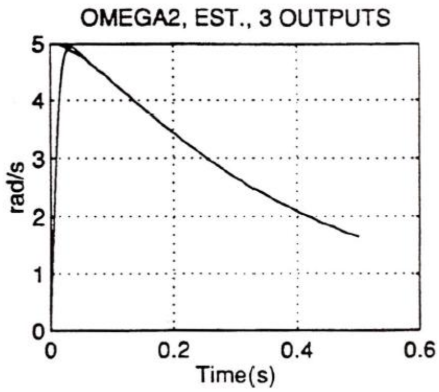
The state is obtained from $\dot{\underline{x}} = A\underline{x}$, the error $\tilde{\underline{x}}$ from $\dot{\tilde{\underline{x}}} = (A - GC)\tilde{\underline{x}}$, with $\tilde{\underline{x}}(0) = \underline{x}(0)$. Finally, $\hat{\underline{x}}(t) = \underline{x}(t) - \tilde{\underline{x}}(t)$

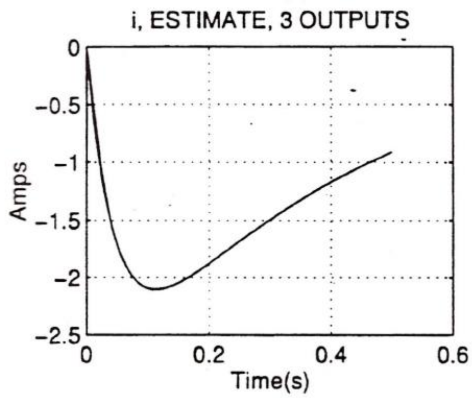
c) Without ω_2, the observer gain is

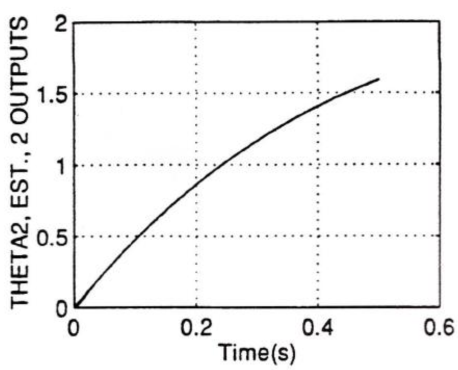
$$G = \begin{bmatrix} -0.08435 & 0.014435 \\ -4.4572 & 4.8859 \\ 31.889 & -10.886 \\ 1984.2 & -915.39 \\ -271.64 & 115.82 \end{bmatrix}$$

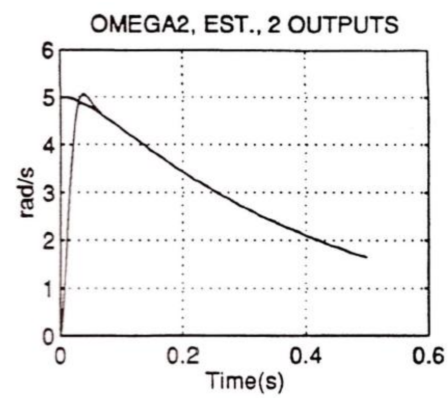
In both cases, the variables θ_2, ω_2, i are estimated as well: there is a marked difference in the estimates of Δ and Ω.

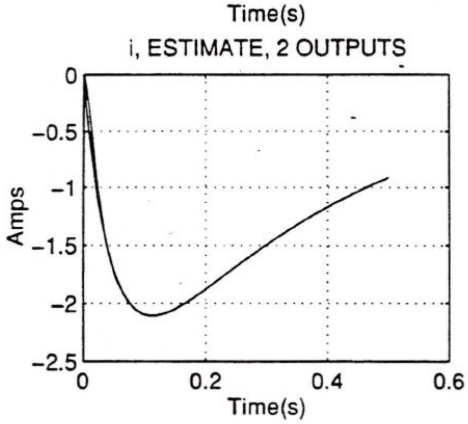

THETA2, ESTIMATE, 3 OUTPUTS
rad
deg
Time(s)


OMEGA2, EST., 3 OUTPUTS
rad/s
Time(s)


i, ESTIMATE, 3 OUTPUTS
Amps
Time(s)


THETA2, EST., 2 OUTPUTS
Time(s)


OMEGA2, EST., 2 OUTPUTS
rad/s
Time(s)


i, ESTIMATE, 2 OUTPUTS
Amps
Time(s)

3. (7.27)

(a) Here Q=0, R=1. MATLAB requires nonzero Q, so try $Q = \epsilon I$ for decreasing ϵ until a limit appears to have been reached.

```
g=9.8; L1=1;L2=0.5; SingularValues=[];
A=[0      0       0      1 0 0
   0      0       0      0 1 0
   0      0       0      0 0 1
   0     -g      -g      0 0 0
   0  2*g/L1   g/L1      0 0 0
   0   g/L2   2*g/L2    0 0 0]
B=[0;  0;  0;  1;  -1/L1; -1/L2]
Ctl=[B, A*B, A^2*B, A^3*B, A^4*B, A^5*B]
rank(Ctl)
Q=zeros(size(A))
R=1
Q=eye(6)
[K,S,E] = lqr(A,B,Q/10000,R)
```

We get from MATLAB

$$K = \begin{pmatrix} 0.0100 & 219.5232 & -217.4130 & 0.2538 & 73.1877 & -46.8427 \end{pmatrix}.$$

(b) Solve for the responses θ_1 and θ_2, and F.

```
newA=A-B*K
newB=[B]
newC=[ 0 1 0 0 0 0; 0 0 1 0 0 0];
newSys=ss(newA,newB,newC,0)

x0=[0 0.1 -0.1 0 0 0]';
tfinal=5;
[y,t,x]=initial(newSys,x0,tfinal);
F=-x*K';
power=F.*x(:,4);
fig=figure;
plot(t,[y -x*K' power]);legend('\theta_1','\theta_2','F','power')
saveas(fig,'p7_27b','epsc')

maxF=max(abs(F))
maxPower=max(abs(power))
```

The max of $|F(t)|$ is 42.8 and max power is 40, as shown in Figure 2.

(c) $l_1 = 1m$ and $l_2 = 0.9m$

We get

$$K = \begin{pmatrix} 0 & 1390.0758 & -1390.0012 & 0.0078 & 447.163 & -418.2536 \end{pmatrix}.$$

The max of $|F(t)|$ is 278 and max power is 1923.6, as shown in Figure 3.

(d) By trial and error, $Q_{11} = 1$ works, for which we find the maximum power to be 45.2w.

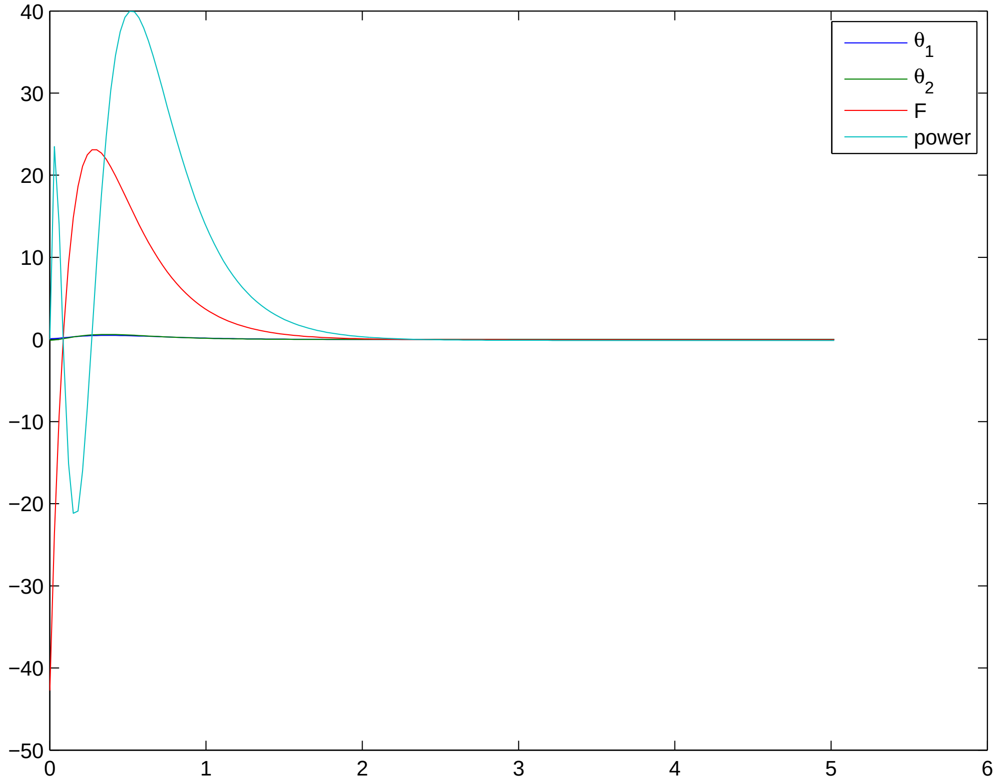


Figure 2: Plot of 7.27 (b).

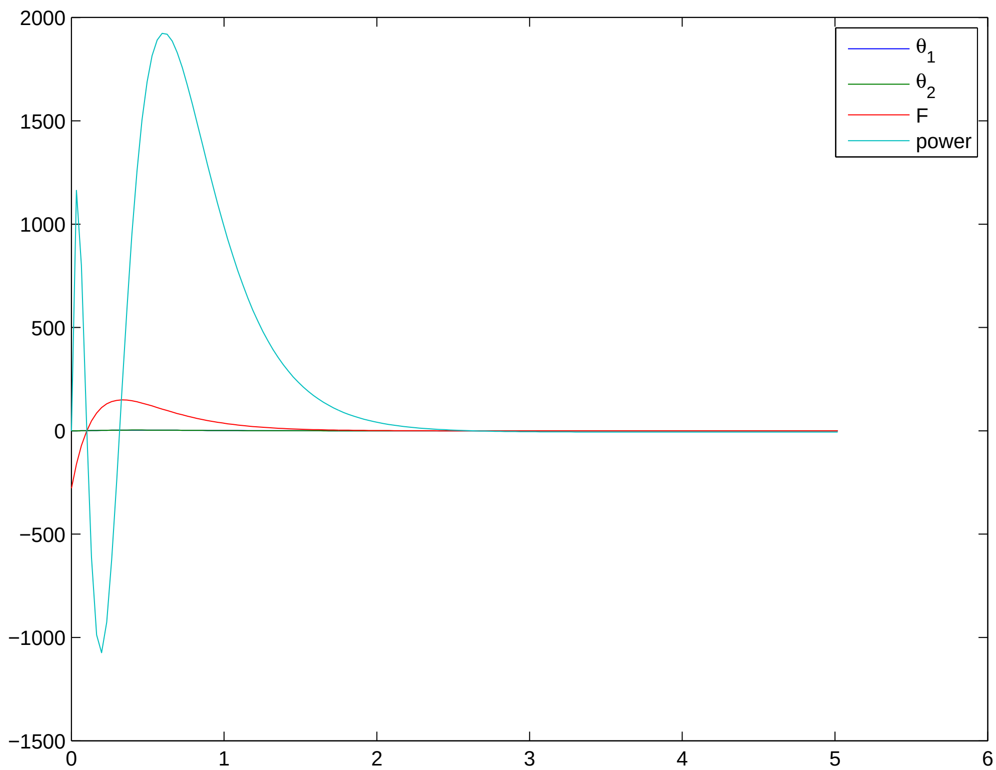


Figure 3: Plot of 7.27 (c).

4. (7.48) MATLAB code

```
g=9.8; L1=1;L2=0.5; SingularValues=[];
A=[0      0       0      1 0 0
   0      0       0      0 1 0
   0      0       0      0 0 1
   0     -g      -g      0 0 0
   0   2*g/L1  g/L1      0 0 0
   0    g/L2   2*g/L2  0 0 0]
B=[0;  0;  0;  1;  -1/L1; -1/L2]
C=[1 0 0 0 0 0
   0 1 0 0 0 0
   0 0 1 0 0 0]
V=0.001^2*eye(3);
w0=1;
W=diag([0 0 0 0 w0^2 w0^2]);
[P,L,Gt] = care(A',C',W,V)
G=Gt';

e_value=eig(A-G*C)
```

Run the codes, we get the following optimal observer gain

```
G=   5.26   -0.0569 -0.0683
    -0.0569   45.161  0.3318
    -0.0683   0.3318  45.608
     13.837  -10.1888 -10.472
     7.2966   1019.81  10.160
     6.9769   19.9565  1040.1

e_value =
      -2.6303 + 2.6319i
      -2.6303 - 2.6319i
     -22.8922 +21.8398i
     -22.8922 -21.8398i
     -22.4923 +22.2299i
     -22.4923 -22.2299i
```

5. (7.68)

(a) The state feedback design is covered in part b) of 7.27.

(b) Observer-based design

The closed-loop is as following

$$\frac{d}{dt}\begin{pmatrix} x \\ \hat{x} \end{pmatrix} = \begin{pmatrix} A & -BK \\ GC & A-GC-BK \end{pmatrix}\begin{pmatrix} x \\ \hat{x} \end{pmatrix}$$

```
%% Define constants
g=9.8; L1=1;L2=0.5; SingularValues=[];
A=[0     0       0      1 0 0
   0     0       0      0 1 0
   0     0       0      0 0 1
   0    -g      -g      0 0 0
   0   2*g/L1  g/L1     0 0 0
   0    g/L2   2*g/L2   0 0 0]
B=[0;  0;  0;  1;  -1/L1; -1/L2]
C=[1 0 0 0 0 0
   0 1 0 0 0 0
   0 0 1 0 0 0]

%% oberver deisgn
V=0.001^2*eye(3);
w0=1;
W=diag([0 0 0 0 w0^2 w0^2]);
[P,L,Gt] = care(A',C',W,V)
G=Gt';

e_value=eig(A-G*C)

%% controler design
Q=zeros(size(A))
R=1
Q=eye(6)
[K,S,E] = lqr(A,B,Q/10^(10),R)

%% simulate response
newA=[A    -B*K
      G*C A-G*C-B*K]
newB=[B;B]
newC=eye(12)
newSys=ss(newA,newB,newC,0)

x0=[0 0.1 -0.1 0 0 0 0 0 0 0 0 0]';
tfinal=2;
[y,t,x]=initial(newSys,x0,tfinal);
plot(t,[y]);
```