10/21 (1)

Recall:

$$\dot{x} = Ax$$

used: $AV = V\Lambda \iff V^{-1}AV = \Lambda$

define: $z := V^{-1}x$

and observe $\dot{z} = V^{-1}\dot{x} = V^{-1}AVz$

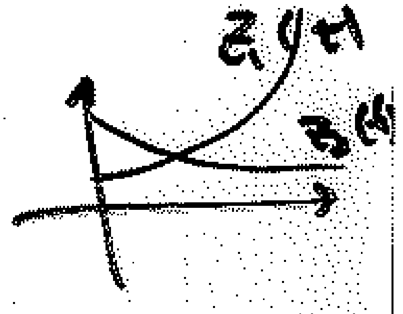
$$\boxed{\dot{z} = \Lambda z}$$

If all eigenvalues of A are real

$$\begin{pmatrix} \dot{z}_1 \\ \dot{z}_2 \end{pmatrix} = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \end{pmatrix}$$

$z_1(t) = e^{\lambda_1 t} z_1(0)$

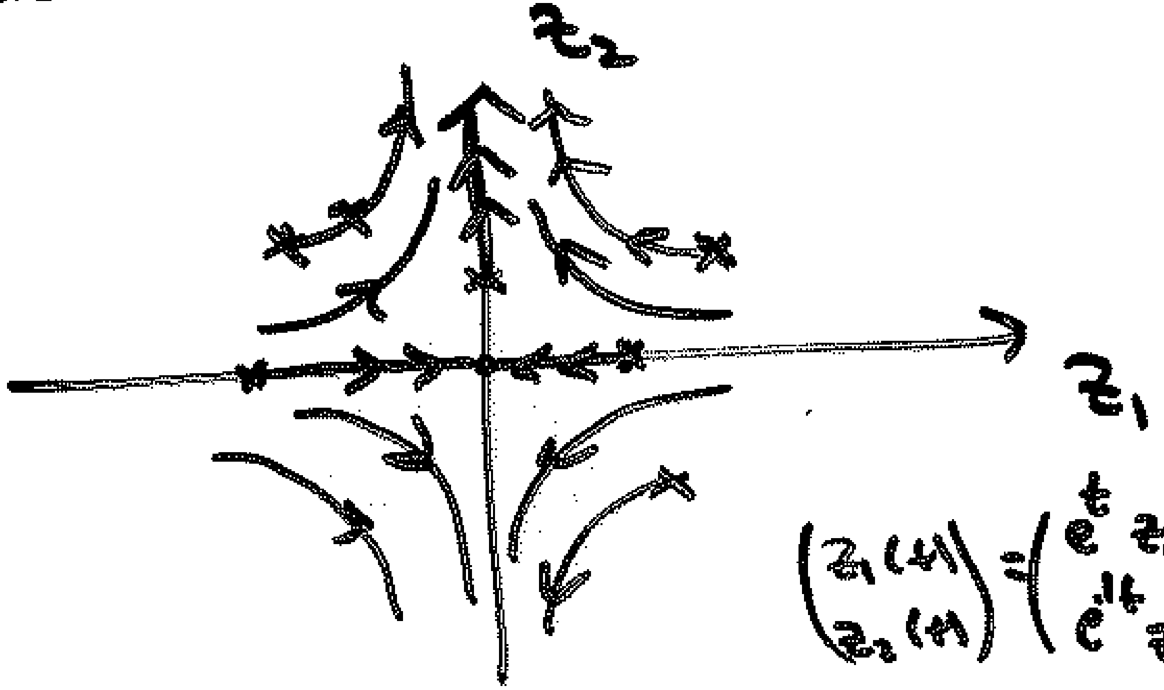
$z_2(t) = e^{\lambda_2 t} z_2(0)$



Another view:

plot "trajectories" in (z_1, z_2) plane

"phase plane"



$$\begin{pmatrix} z_1(t) \\ z_2(t) \end{pmatrix} = \begin{pmatrix} e^{t} z_1(0) \\ e^{.1t} z_2(0) \end{pmatrix}$$

$$\begin{pmatrix} \dot{z}_1 \\ \dot{z}_2 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & +0.1 \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \end{pmatrix}$$

i.e. $\begin{pmatrix} z_1(0) \\ z_2(0) \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \Rightarrow \begin{pmatrix} z_1(t) \\ z_2(t) \end{pmatrix} = \begin{pmatrix} e^{-t} \\ 0 \end{pmatrix}$

$\begin{pmatrix} z_1(0) \\ z_2(0) \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \Rightarrow \begin{pmatrix} z_1(t) \\ z_2(t) \end{pmatrix} = \begin{pmatrix} 0 \\ e^{.1t} \end{pmatrix}$

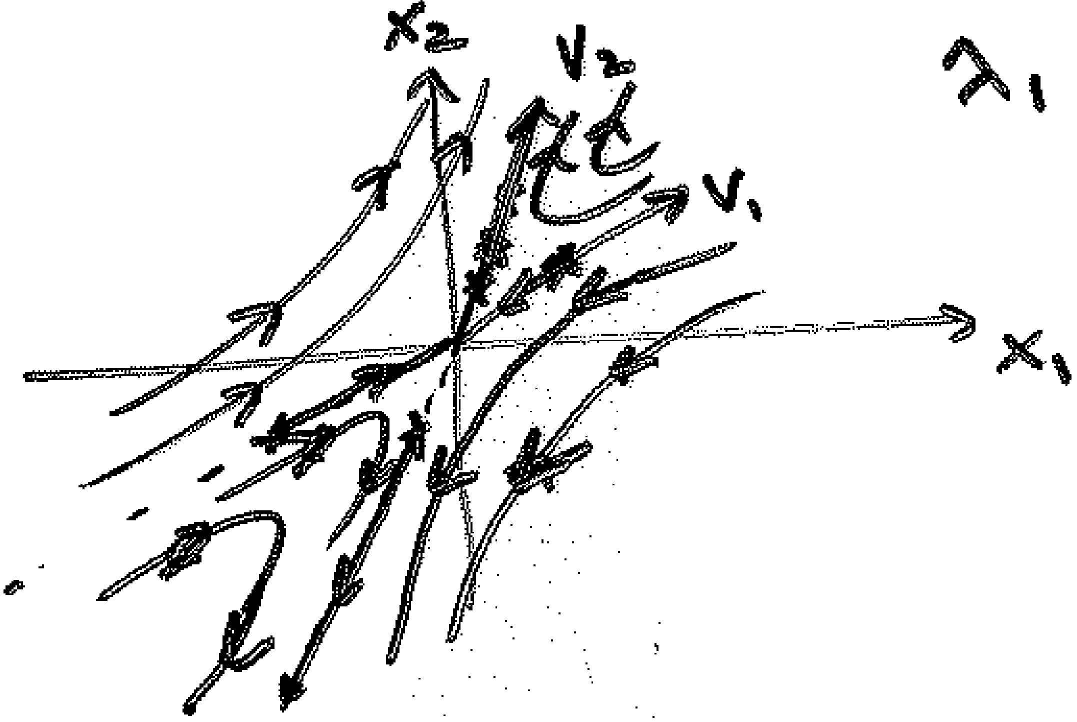
in general

"principal directions"

are the eigenvectors of A

$\lambda_2 > 0$

$\lambda_1 < 0$



4

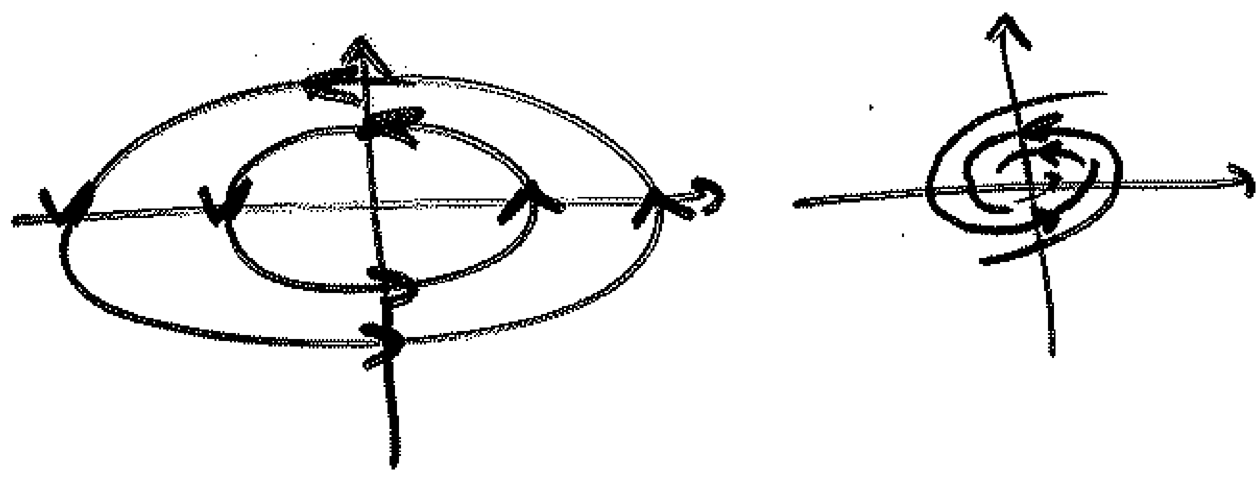
When eigenvalues are complex:

Can diagonalize A with 2x2 real block diagonals corresponding to complex eigenvalues, and scalar blocks corresponding to real eigenvalues:

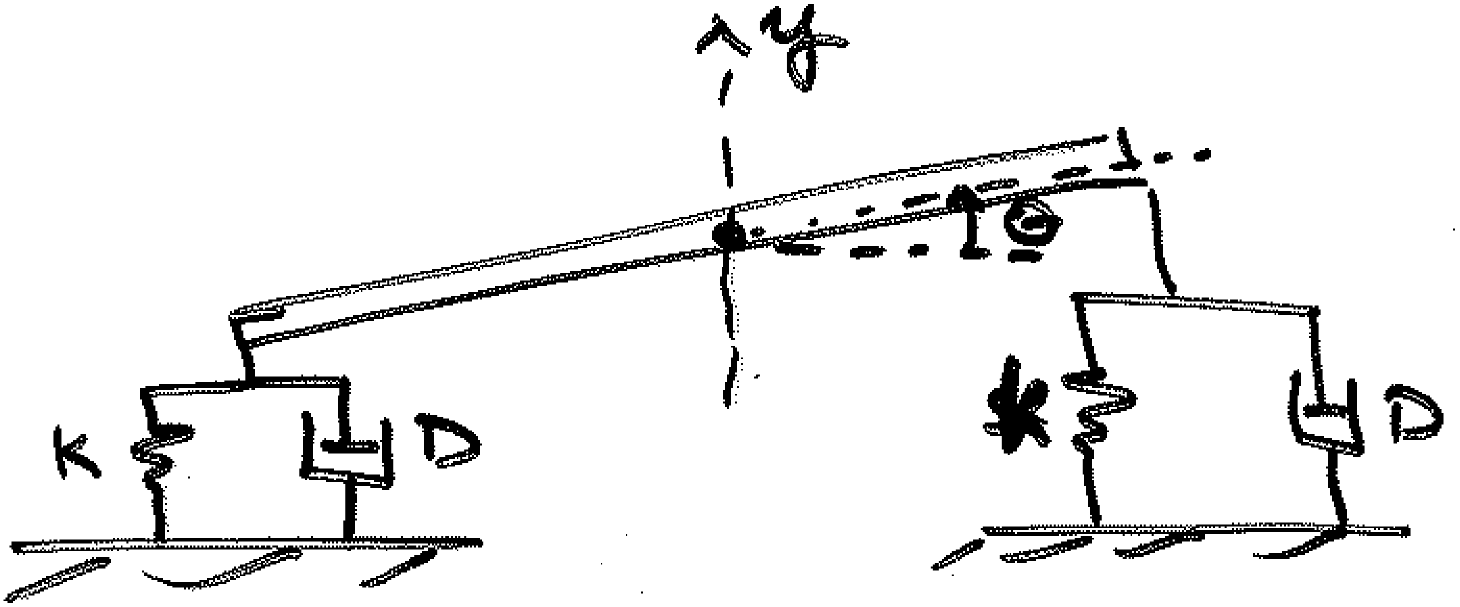
possibly 2x2

$$\begin{bmatrix} A \end{bmatrix}\begin{bmatrix} V \end{bmatrix} = \begin{bmatrix} V \end{bmatrix}\begin{bmatrix} \boxed{\;\;} & & \\ & \boxed{\;\;} & \\ & & \ddots \end{bmatrix}$$

each 2x2 block represents oscillatory dynamics

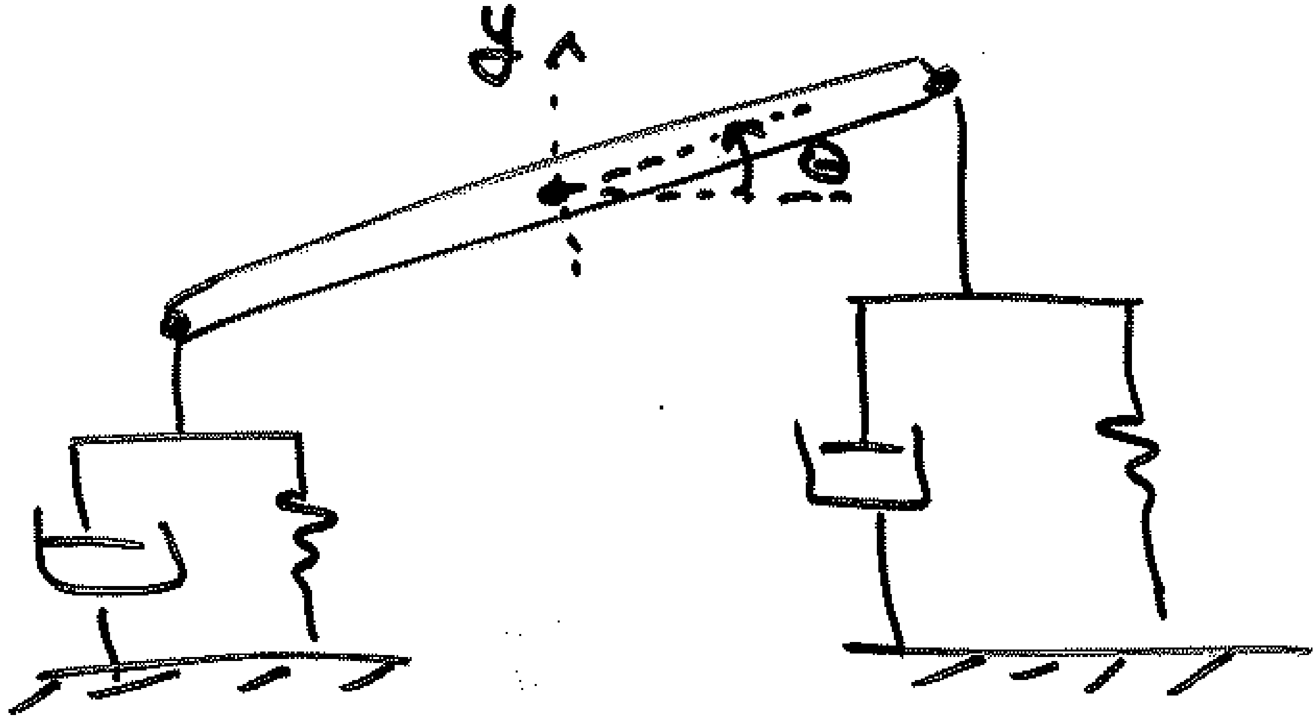


Spring Beam (Belanger Chap. 2)



state variables $\begin{pmatrix} y \\ \theta \\ v \\ \dot{\theta} \end{pmatrix}$

Observability:



$$\dot{x} = A x$$

assume input is zero

$$y = C x$$

measurments (angles, or positions or velocities) for any initial condition $x(0)$

output ~~[illegible]~~ is $\quad x(t) = e^{At} x(0)$

$$y = Cx = Ce^{At} x(0)$$

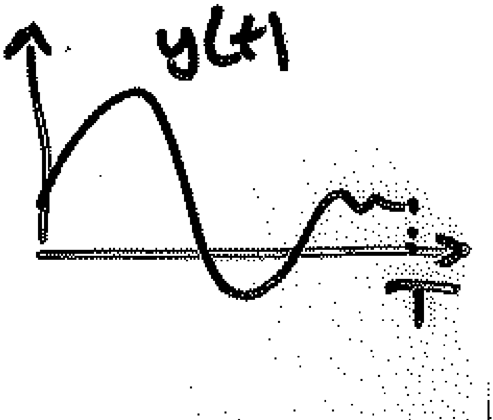
assume $y(t)$ is a scalar

$$y(t) = \begin{bmatrix} & C & \end{bmatrix} \begin{bmatrix} & & \\ & e^{At} & \\ & & \end{bmatrix} \begin{bmatrix} x(0) \\ \\ \\ \end{bmatrix}$$

<u>observability problem:</u>

given the output over a time interval $\quad y(t), \; 0 \le t \le T$

$$\begin{bmatrix} x_1(0) \\ x_2(0) \\ \vdots \\ x_n(0) \end{bmatrix} \longrightarrow$$



(1) A system $\dot{x} = Ax$, $y = Cx$ is observable

if ~~fro~~ the initial condition $x(0)$ can be <u>uniquely determind</u> from the output

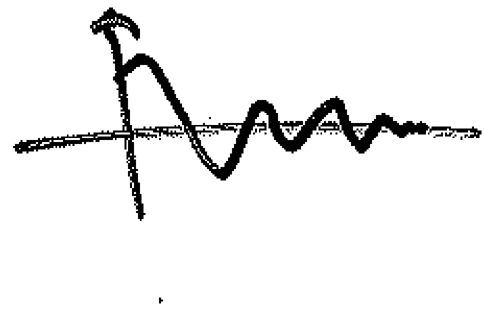
$\{y(t) \; ; \; 0 \le t \le T\}$

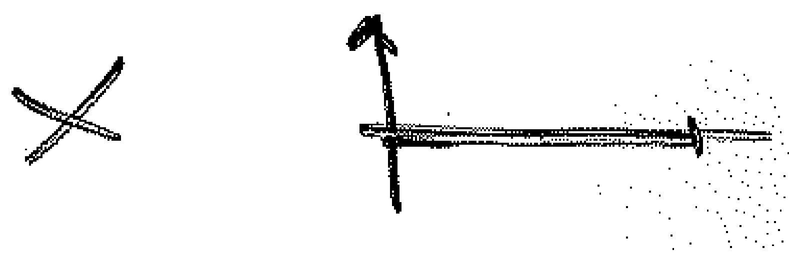
no two i.c. $x_a(0), x_a'(0)$ produce the same output function.

Equivalent to

(2) no initial condition $x(0) \neq 0$

produces $y(t) = 0 \; ; \; 0 \le t \le T$





The test:

form the observability matrix

$$\mathcal{O} := \begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^{n-1} \end{bmatrix} \quad ; \; n \text{ state dimension}$$

system is observable

$$\Updownarrow$$

$\mathcal{O}$ has full rank