Last time

$$\dot{x} = A_c x + B_c u$$

(A_c, B_c) in controllable canon. form

Can design K_c to place eigenvalues of $A_c + B_c K_c$

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given controllable (A, B)

$$\dot{x} = Ax + Bu$$

can find T s.t. $z = Tx$

$$\dot{z} = \underbrace{TAT^{-1}}_{A_c} z + \underbrace{TB}_{B_c} u$$

controllable canonical form

~~Hints:~~

Find K_c s.t. $A_c + B_c K_c$ has desired eigenvalues

feedback $\quad u = K_c z$

$$\therefore \quad u = \underbrace{K_c T}_{\boxed{K := K_c T}} x$$

<u>claim</u>: eigenvalues$(A + BK)$

$$\|$$

eigenvalues$(A_c + B_c K_c)$

<u>check</u>: $\underline{A_c + B_c K_c} = TAT^{-1} + TBKT^{-1}$

$$= T\,(\underline{A + BK})\,T^{-1} \quad \checkmark$$

Similarity Transformations: for any M

eigenvalues of (M) = eigenvalues$(T^{-1}MT)$

Algorithm for Pole Placement:

~~Ste~~ Given (A,B) controllable

1. Find T s.t.
   $A_c = TAT^{-1}$; $B_c = TB$
   (A_c, B_c) in controllable canon. form

2. Design k_c s.t. $A_c + B_c k_c$
   has desired eigenvalues

3. $K = k_c T$

in MATLAB use "place"

How to transform in Cont. Can. Form?

pre req.

Consider $(A,B) = \left(\begin{bmatrix} 0 & 1 & & \\ & & \ddots & \\ & & & 1 \\ \alpha_0 & \cdots & & \alpha_{n-1} \end{bmatrix}, \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \right)$

Is it controllable?

check:

$$\mathcal{C} = \begin{bmatrix} B & AB & \cdots & A^{n-1}B \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 & & 1 \\ \vdots & \vdots & 1 & \iddots & * \\ & & 0 & & * \\ & & 1 & * & \vdots \\ 0 & 1 & * & & * \\ 1 & \alpha_{n-1} & * & & \end{bmatrix}$$ nonsingular

because: upper or lower triang forms

$$\begin{bmatrix} \lambda_1 & & 0 \\ & \ddots & \\ * & & \lambda_n \end{bmatrix}, \quad \begin{bmatrix} \lambda_1 & & * \\ & \ddots & \\ 0 & & \lambda_n \end{bmatrix}$$

lower triangular   upper tri.

$$\therefore \quad \begin{bmatrix} 1 & & 0 \\ & \ddots & \\ * & & 1 \end{bmatrix} \text{ has all eigenvalues at } \underline{1}$$

$$\underbrace{\begin{bmatrix} 1 & & 0 \\ & \ddots & \\ * & & 1 \end{bmatrix}}_{\text{nonsingular}} \underbrace{\begin{bmatrix} 0 & & 1 \\ & \unicode{x22F0} & \\ 1 & & 0 \end{bmatrix}}_{\text{nonsingular}} = \underbrace{\begin{bmatrix} 0 & & 1 \\ & \unicode{x22F0} & \\ 1 & & * \end{bmatrix}}_{\text{nonsingular}}$$

nonsingular $\Rightarrow$ nonsingular

Sec. 7.3

Given A, B controllable,

(1) form $\mathcal{C} = [B \ AB \cdots A^{n-1}B]$

(A,B) controllable $\iff$ $\mathcal{C}$ nonsingular

(2) form $\mathcal{C}_c = [B_c \ A_cB_c \cdots A_c^{n-1}B_c]$

to find B_c, A_c, find transfer function from A, B, or just the char. polynomial of A.

(3) $H = \mathcal{C}\,\mathcal{C}_c^{-1}$ $\boxed{\begin{array}{c} H = T^{-1} \\ T \text{ as before} \end{array}}$

then $A_c = H^{-1}AH$; $B_c = H^{-1}B$ will be in canonical form.

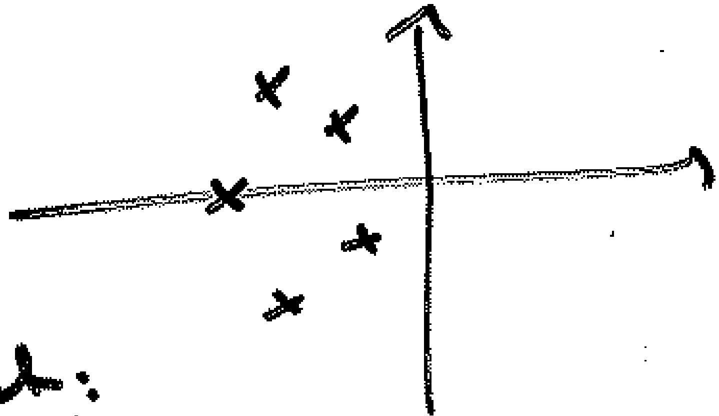
Discussion of pole placement

$\dot{x} = Ax + Bu$: open loop

$u = kx$: feedback

$\dot{x} = (A+BK)x$: closed loop

$$\boxed{\text{Designer needs to choose locations of c.l. eigenvalues } \lambda_1, \ldots, \lambda_n}$$



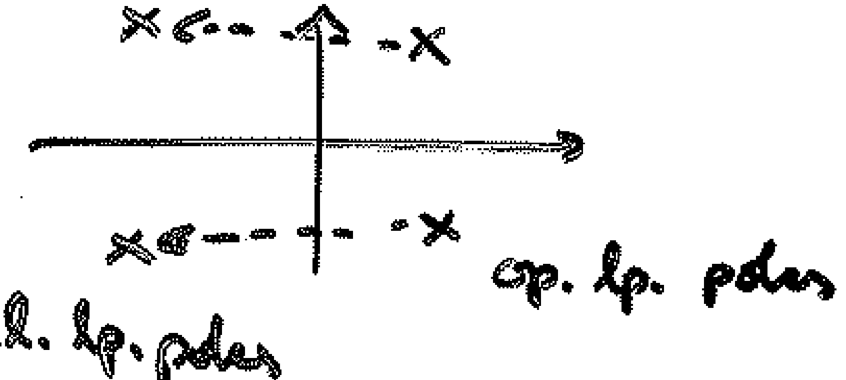
pole Placement:

$$(\lambda_1, \ldots, \lambda_n) \xrightarrow{\text{pole placement}} [k_1, \ldots, k_n]$$

: tradeoffs:

1 $Re(\lambda)$'s ~~determines~~ determines settling time ~~&~~ or speed of regulation

2



roughly speaking

the further the poles are moved, the larger the feedback gains $k_1, \ldots, k_n$

This produces larger control signals

$$u(t) = K\ x(t)$$