* Get working on HW6, due Tu @ 8 PM

* Office hours 11-1 on Tu

Summary of results on Realizations:

1 Given $\dot{x} = Ax + Bu$, $y = Cx$ denoted (A,B,C)

the T.F. from u to y is

$$H(s) = C(sI-A)^{-1}B$$

2 The "dimension" of a realization is the # of state variables (dimension of the state vector x)

2.5 A realization that is both cont. & obs. is called "minimal"

3 The "order" of a T.F.

$$H(s) = \frac{b_n s^n + \cdots + b_0}{s^n + \cdots + a_0}$$ is $\underline{\underline{n}}$

(the degree of the denominator) (num, den have no common factors)

our systems:

deg (numerator) $\leq$ deg (denominator)

(proper systems)

Theorem: $H(s) = C(sI-A)^{-1}B$

1. $\text{order}(H(s)) \leq \dim(A)$

2. (A,B) controllable & (C,A) obs. (i.e. minimal) then
$$\text{order}(H(s)) = \dim(A)$$

3. ~~Realization~~ Given any T.F. $H(s) = \frac{p(s)}{q(s)}$ with polynomials p & q coprime, can always find a minimal realization of dimension equal to order(q)

Thm

let (A_1, B_1, C_1) and (A_2, B_2, C_2) be two minimal realizations of the same T.F.

so

$$C_1 (sI - A_1)^{-1} B_1 = C_2 (sI - A_2)^{-1} B_2$$

Then there exists a transformation T s.t.

$$A_2 = T^{-1} A_1 T$$
$$B_2 = T^{-1} B_1$$
$$C_2 = C_1 T$$

i.e. the two realization are similar

<u>Exercise</u>: Given two minimal realizations

(A_1, B_1, C_1) and (A_2, B_2, C_2)

of the same T.F.

<u>Find the similarity transformation between them</u>

i.e. find T s.t.

$$A_2 = T^{-1} A_1 T$$

$$B_2 = T^{-1} B_1$$

$$C_2 = C_1 T$$

but observe

$$\mathcal{C}_2 = [B_2 \;\; A_2 B_2 \cdots A_2^{n-1} B_2]$$

$$= [T^{-1}B_1 \;\; T^{-1}A_1 T T^{-1} B_1 \; \cdots \; T^{-1}A_1 T T^{-1} A_1 T \cdots T^{-1} B_1]$$

$$[T^{-1}B_1 \;\; T^{-1}A_1 B_1 \cdots T^{-1} A_1^{n-1} B_1]$$

$$\mathcal{C}_2 = T^{-1}[B_1 \cdots A_1^{n-1}B_1]$$

$$\therefore \mathcal{C}_2 = T^{-1}\mathcal{C}_1$$

then $\boxed{T = \mathcal{C}_1\mathcal{C}_2^{-1}}$

we wrote this last time

to go from a controllable realization

to controllable canonical

form.

Output Regulation (regulation to a nonzero set point)

Last time learned to design K

s.t. given $\dot{x} = Ax + Bu$
$y = Cx$

the feedback $u = Kx$

gives:
$$\dot{x} = (A+BK)x$$
$$y = Cx$$

if $(A+BK)$ is stable, then

$$X(t) \xrightarrow{t \to \infty} 0$$ (for any i.c. $x(0)$)

Consequently

$$y(t) \xrightarrow{t \to \infty} 0$$ "regulation to zero"

How about: Design so that $y(t) \xrightarrow{t \to \infty} y^*$ **Quickly**:

Control needs to be of the form

$$u(t) = u^* + \underbrace{k\left(x(t) - x^*\right)}$$

feed forward

feedback control

u^* is the input needed to y^* as output in steady state, corresps to x^* (in steady state)

u^*, y^*, x^* must satisfy

$$0 = \dot{x} = A x^* + B u^*$$

$$y^* = C x^*$$

known

to be solved for

$$\begin{bmatrix} A & B \\ C & 0 \end{bmatrix} \begin{bmatrix} x^* \\ u^* \end{bmatrix} = \begin{bmatrix} 0 \\ y^* \end{bmatrix}$$

in $$\begin{bmatrix} x^* \\ u^* \end{bmatrix} = \begin{bmatrix} M_1 \\ M_2 \end{bmatrix} y^*$$

$$u = u^* + k\,(\underline{x - x^*})$$