11/18 ①

- No class next Tuesday 11/25

- HW # 7 will be posted later today
Due Thu 12/5.
longer than normal.
will count as 1.5 x regular HW
Cannot be dropped from HW av.

State Observers (Estimator)

- of interest by them selves, observe (measure) output, then use that to dynamically estimate the entire state.

- use as building block in design of "output controllers"

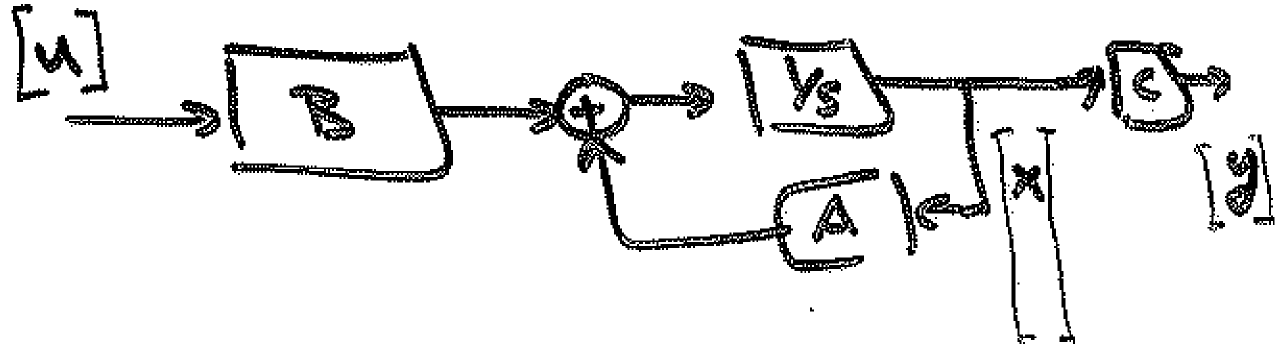
Output Controller: feed back controller that uses an output rather than the full state.

Observers:

Given $\dot{x} = Ax + Bu$

$y = Cx$

u, y : measured signals



find x from u, y.

- If given only one time measurement

$u(\bar{t}), y(\bar{t})$

determine $x(\bar{t})$?

Can not be done with single measurment, but collecting (4)

$\{u(t) ; 0 \leq t \leq T\}$

$\{y(t) ; 0 \leq t \leq T\}$

can then determine $x(T)$

using knowledge of dynamics A, B, C.

Thought experiment:

Real system: $\dot{x} = Ax + Bu$
$y = Cx$
$; x(0) = ?$

build a simulator

$$\dot{\hat{x}} = A\hat{x} + Bu$$

run the simulator

Observe:

* if simulator starts at correct initial state, i.e. $\hat{x}(0) = x(0)$

then $\hat{x}(t) = x(t)$

for $t \geq 0$

* will not work in reality because it behaves badly in the presence of small errors in $A, B, u, x(0)$

* Didn't make use of the measurement $y(t)$!

A better Scheme:

$$\underbrace{\dot{\hat{x}} = A\hat{x} + Bu}_{\text{system model}} + L\,\underbrace{(y-\hat{y})}$$

$\hat{y} = C\hat{x}$

predicted output

observer gain (to be designed)

prediction error

Investigate how well scheme works:

Define error $e(t) := x(t) - \hat{x}(t)$

note: $e(0) = x(0) - \hat{x}(0)$

mismatch in initial conditions

usually $e(0) \neq 0$.

Can we say: $e(t) \xrightarrow{t \to \infty} 0$

$\boxed{?}$

Find the "Dynamics" of $e(t)$

Error Dynamics: $e := x - \hat{x}$

$$\dot{e} = \dot{x} - \dot{\hat{x}} = Ax + \cancel{Bu} - A\hat{x} - \cancel{Bu} - L(Cx - C\hat{x})$$

$$= A(x - \hat{x}) - LC(x - \hat{x})$$

Error Dynamics:

$$\boxed{\dot{e}(t) = (A-LC)\, e(t)}$$

$$e(t) \xrightarrow{t \to \infty} 0$$

$$\Updownarrow$$

$(A-LC)$ is stable (all eigs in LHP)

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1. Given $\begin{bmatrix} A \end{bmatrix}$, $\begin{bmatrix} C \end{bmatrix}$ design $L = \begin{bmatrix} \ \end{bmatrix}$

such that $\begin{bmatrix} A \end{bmatrix} - \begin{bmatrix} L \end{bmatrix}\begin{bmatrix} C \end{bmatrix}$ stable

Compare with

2. Given $\begin{bmatrix} A \end{bmatrix}$, $\begin{bmatrix} B \end{bmatrix}$ design $\begin{bmatrix} K \end{bmatrix}$

s.t. $\begin{bmatrix} A \end{bmatrix} + \begin{bmatrix} B \end{bmatrix}\begin{bmatrix} K \end{bmatrix}$ is stable

Given "place" routine

$$(A-LC)^T = \underset{\text{"A"}}{A^T} - \underset{\text{"B"}}{C^T}\,\underset{\text{"k"}}{L^T}$$

Find k s.t. eig(A^T, C^T) are placed i.e. $(A^T + C^T k)$ are placed

then set $L = -k^T$

<u>meaning of pole locations</u>

poles of $A-LC$ determine rate of convergence of error to zero

$$\dot{e} = (A-LC)e$$