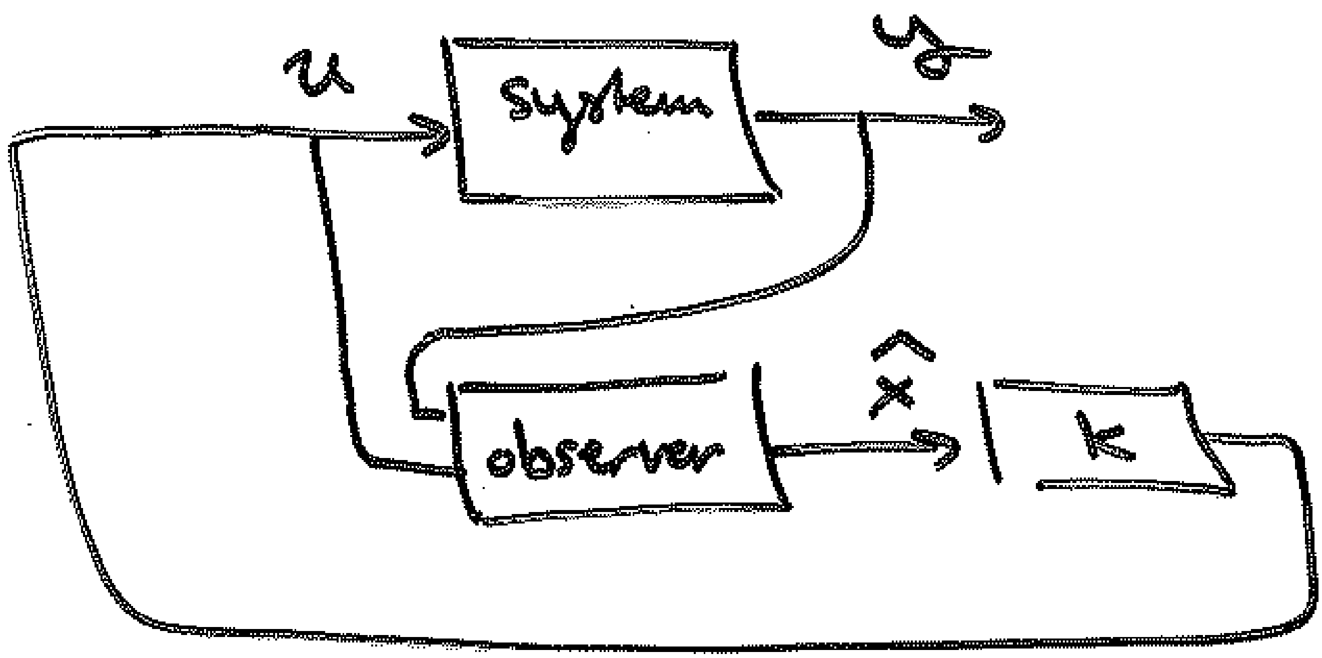
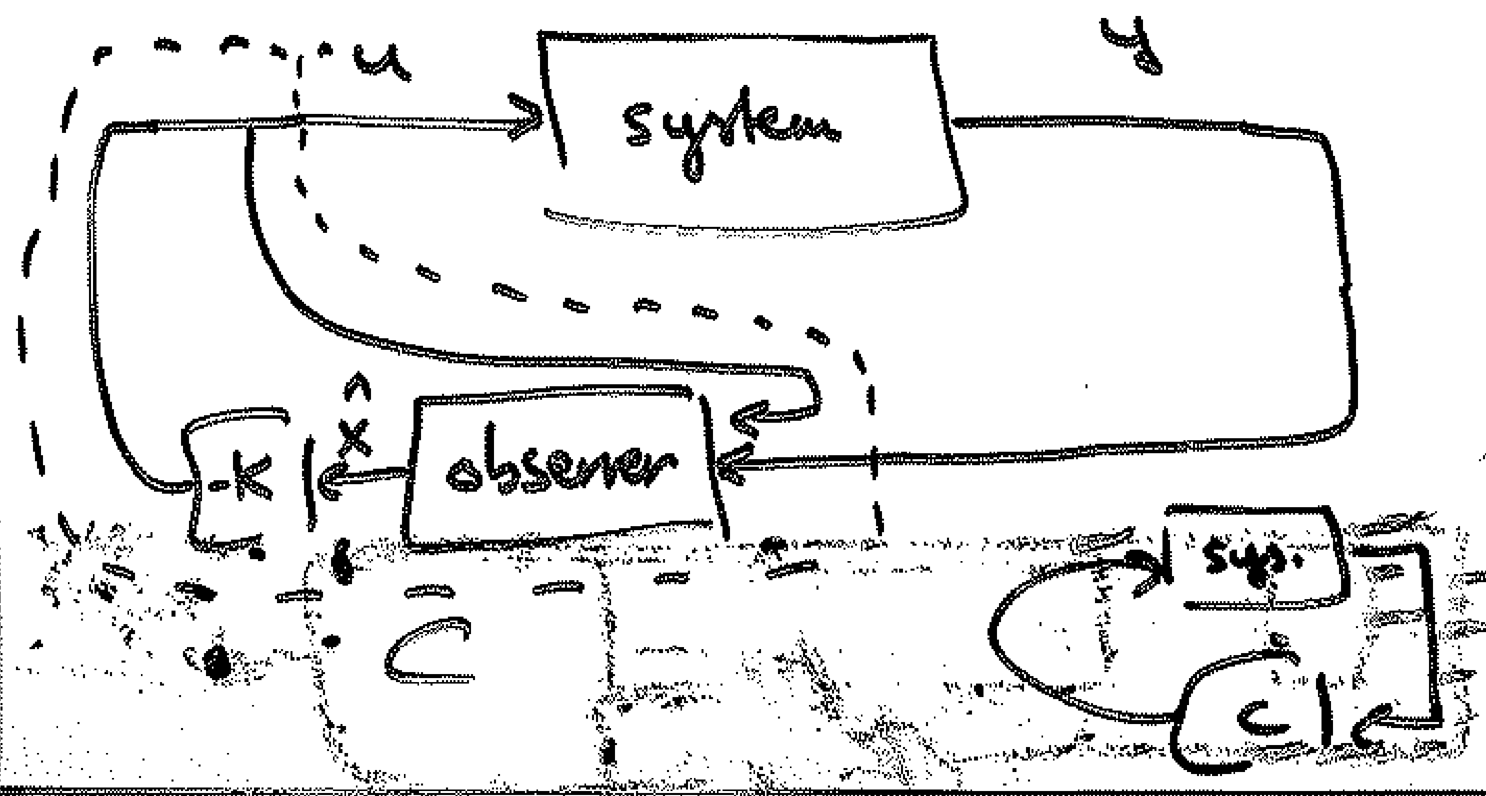
Observer based Controller (1/20) (1)



draw differently



Equations of Controller ②

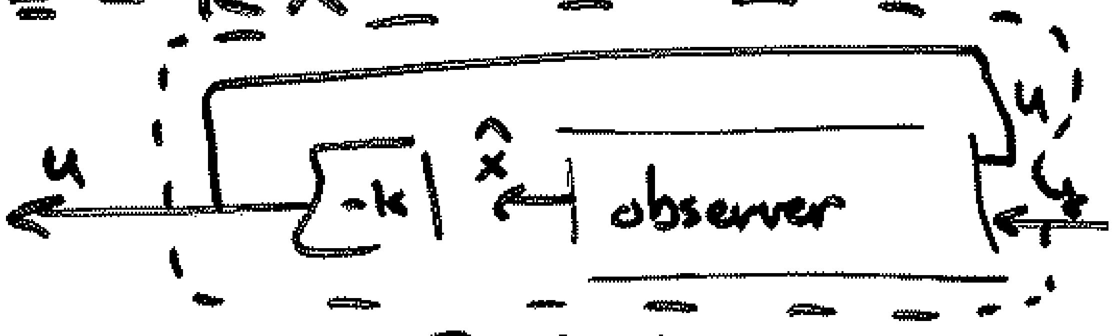
state equation of controller

$$\dot{\hat{x}} = A\hat{x} + Bu + L(y - \hat{y})$$

$$\boxed{\dot{\hat{x}} = (A - LC)\hat{x} + Bu + Ly}$$

& control is generated by

$$u = -k\hat{x}$$



Controller

$$\therefore \quad \dot{\hat{x}} = (A - LC)\hat{x} - \underline{Bk\hat{x}} + Ly$$

observer based control:

$$\dot{\hat{x}} = \boxed{(A - LC - Bk)}\,\hat{x} + \boxed{L}\,y$$

$$u = \boxed{-k}\,\hat{x}$$

Principle behind Observer based Controller

1. Design observer with stable error dynamics to produce $\hat{x}$

2. use $u = -k\hat{x}$

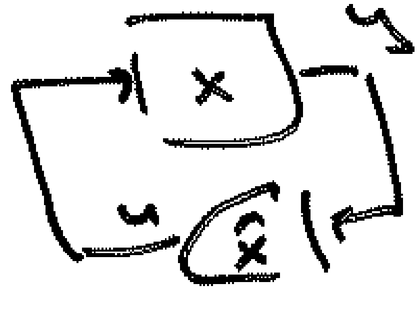
   i.e we state feed back on estimate $\hat{x}$ rather than actual state.

Why does this work?

Seperation Principle:

System equations

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx \end{cases}$$



Controller

$$\cancel{\dot{\hat{x}} = A\hat{x} + B}$$

$$\dot{\hat{x}} = (A - LC - BK)\,\hat{x} + L y$$

$$u = -K\hat{x}$$

c.l.

$$\begin{pmatrix} \dot{x} \\ \dot{\hat{x}} \end{pmatrix} = \begin{pmatrix} A & -BK \\ LC & A-BK-LC \end{pmatrix} \begin{pmatrix} x \\ \hat{x} \end{pmatrix}$$

not easy to see if stable

rewrite using

$$\begin{pmatrix} x \\ e \end{pmatrix} := \begin{pmatrix} x \\ x - \hat{x} \end{pmatrix} = \begin{pmatrix} I & 0 \\ I & -I \end{pmatrix} \begin{pmatrix} x \\ \hat{x} \end{pmatrix}$$

the state equations now are

$$\left(\begin{array}{c} \dot{x} \\ \hdashline \dot{e} \end{array}\right) = \underbrace{\left(\begin{array}{c:c} A-Bk & Bk \\ \hdashline \boxed{0} & A-LC \end{array}\right)}_{A_{cl}} \left(\begin{array}{c} x \\ \hdashline e \end{array}\right)$$

$$\text{eig}(A_{cl}) = \text{eig}(A-Bk) \cup \text{eig}(A-LC)$$

eigenvalues of state feed back problem

eigenvalues of observer error