Ricatti Equations (Algebraic) 12/10

A matrix equation of the form

$$MX + XM^T - XSX + T = 0$$

X: unknown $n \times n$ matrix

M, S, T: given $n \times n$ matrices

* ① Matrix generalization of the quadratic equation

② n^2 coupled system of quadratic equations: like

$$m_{35} x_{25} + x_{52} m_{53} + x_{13} x_{56} s_{12} + T_{35} = 0$$

in n^2 variables.

Solution of LQR Problem: (2)

$$\dot{x} = Ax + Bu$$

$$J = \int_0^\infty x^T Q x + u^T R u \, dt$$

Thm solution $u = kx$

optimal $k = -R^{-1} B^T P$

where P is the (stabilizing) solution of the ARE

$$A^T P + PA - P B R^{-1} B^T P + Q = 0$$

stabilizing means: $A + Bk$ is stable

Matlab: "care"

Continuous-time Algebraic Ric. Eq.

Optimal Observer:

$$\dot{x} = Ax + Bu + w \quad \leftarrow \text{process } \text{~~noise~~ disturbance}$$

$$y = Cx + v \quad \leftarrow \text{measurement noise}$$

Covariance $(w) = W$

Covariance $(v) = V$

optimal observer gain $L = PC^T V^{-1}$

where P is the (stabilizing) solution to the Estimation ARE

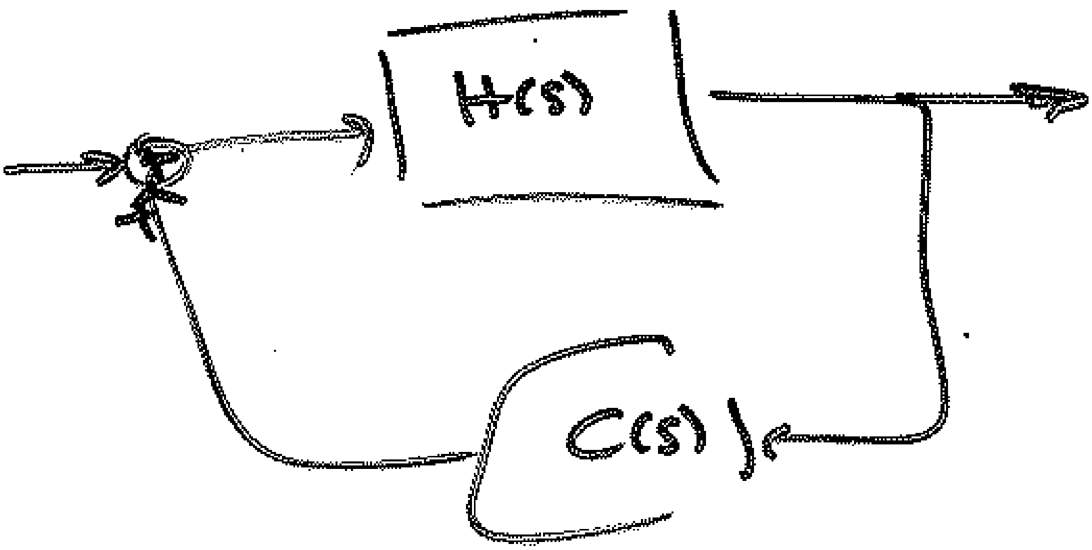
$$AP + PA^T - P\underline{C^T V^{-1} C}P + W = 0$$

stabilizing means: $(A - LC)$ stable

i.e. stable error dynamics

Question:

Given
$$H(s) = \frac{s^2+1}{s^7+5s^5-2s+1}$$



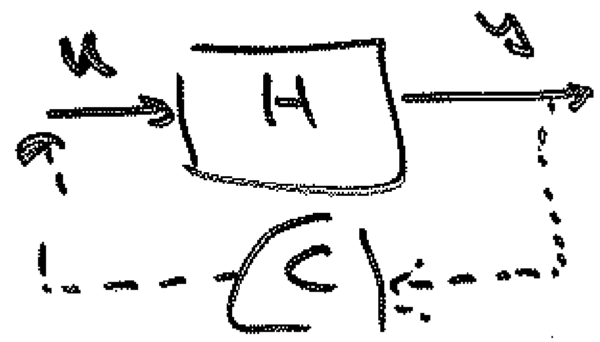
* Find a Controller such that the closed loop system is stable.

$$\frac{H}{1-HC}$$

One Procedure :

1. take $H(s)$ find a state space realization of H

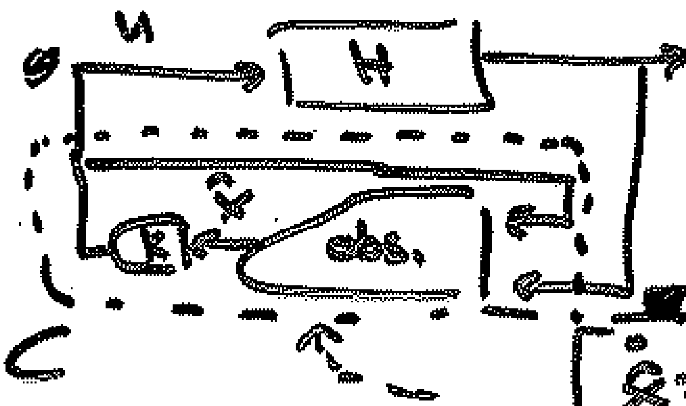
$$\dot{x} = Ax + Bu$$
$$y = Cx$$

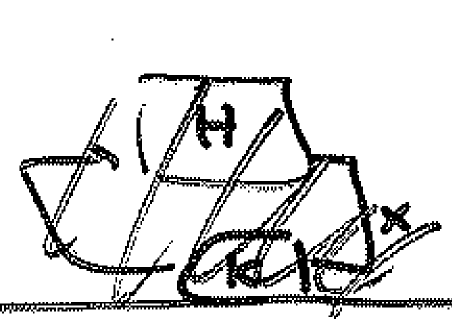


2. Find k s.t. $A+Bk$ is stable

3. Find L s.t. $A-LC$ is stable

4. Construct an observer-based Controller





$$\boxed{\begin{aligned}\dot{\hat{x}} &= (A+Bk-LC)\,\hat{x} + Ly \\ u &= k\,\hat{x}\end{aligned}}$$

observe:

* Observer based $C(s)$'s of the same order as $H(s)$

* Is $C(s)$ stable?
don't know!

$C(s)$ could be either stable or unstable