Wall-Bounded Shear Flow Transition

Beyond Classical Stability Theory

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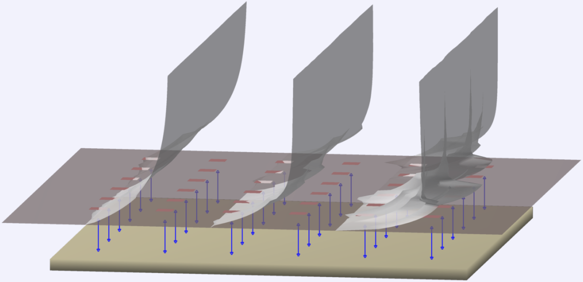
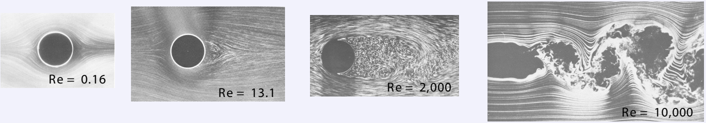


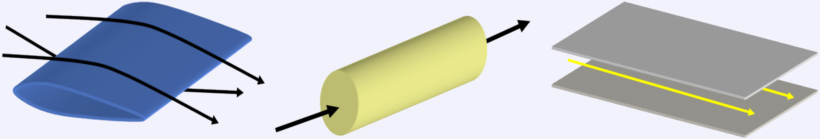
Transition & Turbulence as Natural Phenomena

All fluid flows *transition* (as $0 \xrightarrow{R} \infty$) from laminar to turbulent flows

- Bluff bodies — dominant phenomenon: *separation*

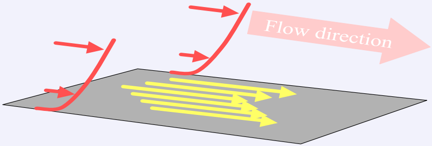


- Streamlined bodies — dominant phenomenon: *friction with walls*

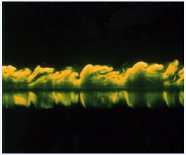


wall-bounded shear flows

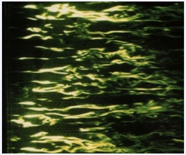
Boundary Layer Turbulence



boundary layer turbulence



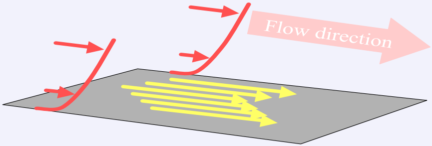
side view



top view

- Transition & Turbulence in Boundary Layer and Channel Flows

Boundary Layer Turbulence



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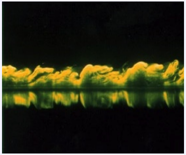

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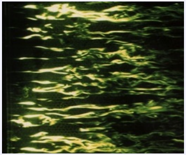
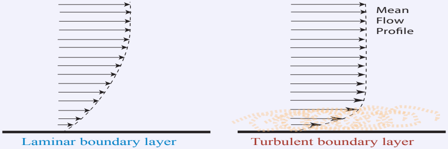
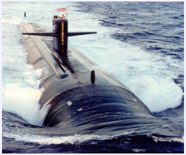


top view



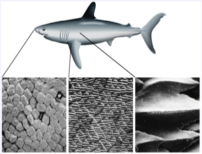
skin-friction drag: laminar vs. turbulent

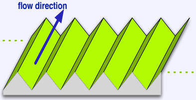


- Transition & Turbulence in Boundary Layer and Channel Flows
- Technologically Important: *Skin-Friction Drag*

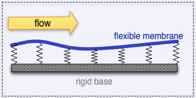
Control of Boundary Layer Turbulence

in nature: *passive control*



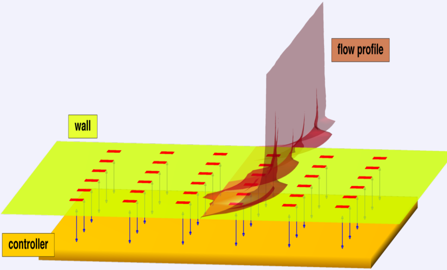


corrugated skin



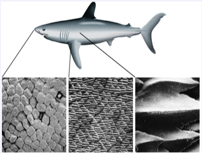
compliant skin

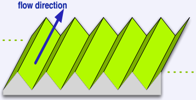
active control with sensor/actuator arrays



Control of Boundary Layer Turbulence

in nature: *passive control*





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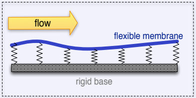

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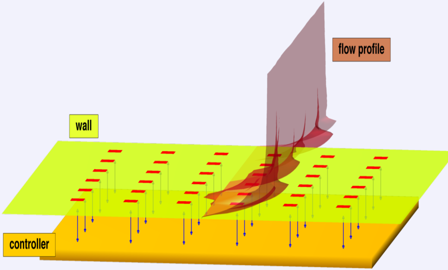


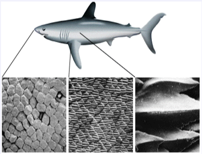
- Intuition: must have ability to actuate at spatial scale comparable to flow structures

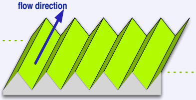
$$\text{spatial-bandwidth of controller} \quad \geq \quad \text{plant's bandwidth}$$

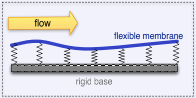
Control of Boundary Layer Turbulence

in nature: *passive control*

active control with sensor/actuator arrays

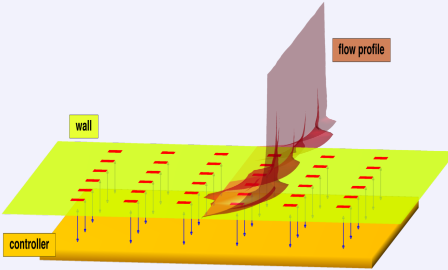






corrugated skin

compliant skin



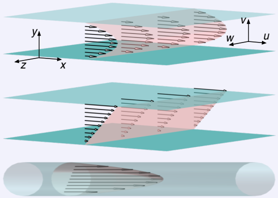
- Intuition: must have ability to actuate at spatial scale comparable to flow structures

  spatial-bandwidth of controller $\geq$ plant's bandwidth

- Caveat: *Plant's dynamics are not well understood* :(

  obstacles $\left\{ \begin{array}{l} \text{not only device technology} \\ \text{also: } \textit{dynamical modeling} \text{ and } \textit{control design} \end{array} \right.$

The Navier-Stokes (NS) equations:

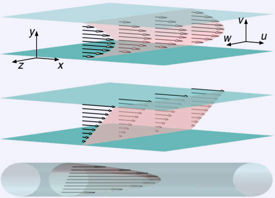


$$\begin{aligned} \partial_t \mathbf{u} &= -\nabla_{\mathbf{u}}\mathbf{u} - \text{grad}\, p + \frac{1}{R}\Delta\mathbf{u} \\ 0 &= \text{div}\, \mathbf{u} \end{aligned}$$

- Hydrodynamic Stability: view NS as a dynamical system
- *laminar flow* $\bar{\mathbf{u}}_R :=$ a stationary solution of the NS equations (an *equilibrium*)

Mathematical Modeling of Transition: Hydrodynamic Stability

The Navier-Stokes (NS) equations:



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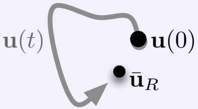
- *laminar flow* $\bar{\mathbf{u}}_R :=$ a stationary solution of the NS equations (an *equilibrium*)

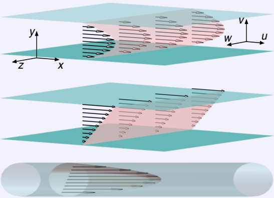
laminar flow $\bar{\mathbf{u}}_R$ stable $\longleftrightarrow$ i.c. $\mathbf{u}(0) \neq \bar{\mathbf{u}}_R$, $\mathbf{u}(t) \xrightarrow{t \to \infty} \bar{\mathbf{u}}_R$

- typically done with dynamics linearized about $\bar{\mathbf{u}}_R$
- various methods to track further "non-linear behavior"



Mathematical Modeling of Transition: Hydrodynamic Stability

The Navier-Stokes (NS) equations:



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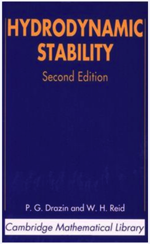
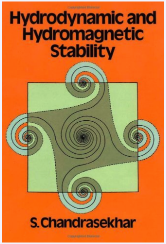
- Hydrodynamic Stability: view NS as a dynamical system
- A very successful (*phenomenologically predictive*) approach for many decades





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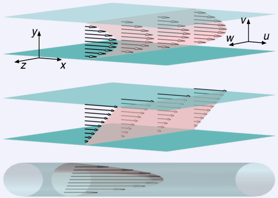

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- Hydrodynamic Stability: view NS as a dynamical system
- however: problematic for wall-bounded shear flows

Flow type	Classical linear theory R_c	Experimental R_c
Channel Flow	5772	$\approx$ 1,000-2,000
Plane Couette	∞	$\approx$ 350
Pipe Flow	∞	$\approx$ 2,200-100,000

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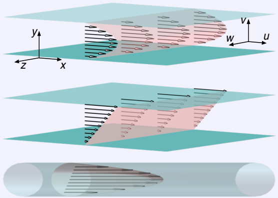

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- was widely believed: *this theory fails because it is* linear
  and "nonlinear effects" are important even for infinitesimal i.c.
- *however, since 90's: story is actually more interesting than that*

Nonmodal Stability Theory, *Schmid, ARFM '07*

- Decompose the fields as $$\mathbf{u} = \underset{\text{laminar}}{\underset{\uparrow}{\bar{\mathbf{u}}_R}} + \underset{\text{fluctuations}}{\underset{\uparrow}{\tilde{\mathbf{u}}}}$$
- Fluctuation dynamics: In *linear* hydrodynamic stability, $-\nabla_{\tilde{u}}\tilde{u}$ is ignored

$$\begin{aligned} \partial_t \tilde{\mathbf{u}} &= -\nabla_{\bar{\mathbf{u}}_R}\tilde{\mathbf{u}} - \nabla_{\tilde{\mathbf{u}}}\bar{\mathbf{u}}_R - \text{grad}\,\tilde{p} + \frac{1}{R}\Delta\tilde{\mathbf{u}} - \nabla_{\tilde{u}}\tilde{u} \\ 0 &= \text{div}\,\tilde{\mathbf{u}} \end{aligned}$$

Mathematical Modeling of Transition: Linearized Stability

- Decompose the fields as

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- Linearization in “Evolution Form”

$$\begin{aligned} \partial_t \begin{bmatrix} \Delta\tilde{v} \\ \tilde{\omega} \end{bmatrix} &= \begin{bmatrix} U''\partial_x - U\Delta\partial_x + \frac{1}{R}\Delta^2 & 0 \\ -U'\partial_z & -U\partial_x + \frac{1}{R}\Delta \end{bmatrix} \begin{bmatrix} \tilde{v} \\ \tilde{\omega} \end{bmatrix} \\ &=: \begin{bmatrix} \mathcal{L} & 0 \\ \mathcal{C} & \mathcal{S} \end{bmatrix} \begin{bmatrix} \tilde{v} \\ \tilde{\omega} \end{bmatrix} =: \mathcal{A} \begin{bmatrix} \tilde{v} \\ \tilde{\omega} \end{bmatrix} \end{aligned}$$

$\tilde{v}$:= wall-normal velocity

$\tilde{\omega}$:= wall-normal vorticity

Mathematical Modeling of Transition: Linearized Stability

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$\tilde{v}$:= wall-normal velocity

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- Classical Linear Hydrodynamic Stability:

Transition $\longleftrightarrow$ Instability $\Leftrightarrow$ $\mathcal{A}$ has spectrum in right half plane

The existence of “exponentially growing normal modes” (of $e^{t\mathcal{A}}$)

The Eigenvalue Problem

- For parallel channel flows
  $\mathcal{A}$ is *translation invariant* in x, z, $\Rightarrow$ Fourier transform in x and z:

$$\frac{\partial}{\partial t}\begin{bmatrix}\hat{v}\\ \hat{\omega}\end{bmatrix} = \begin{bmatrix} ik_x\Delta^{-1}U'' - ik_x\Delta^{-1}U\Delta + \frac{1}{R}\Delta^{-1}\Delta^2 & 0 \\ -ik_zU' & -ik_xU + \frac{1}{R}\Delta \end{bmatrix}\begin{bmatrix}\hat{v}\\ \hat{\omega}\end{bmatrix}$$

- k_x, k_z: spatial frequencies in x, z directions (wave-numbers).

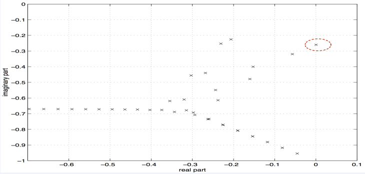
$$\frac{\partial}{\partial t}\begin{bmatrix}\hat{v}(t,k_x,.,k_z)\\ \hat{\omega}(t,k_x,.,k_z)\end{bmatrix} = \hat{\mathcal{A}}(k_x,k_z)\begin{bmatrix}\hat{v}(t,k_x,.,k_z)\\ \hat{\omega}(t,k_x,.,k_z)\end{bmatrix}$$

Essentially:

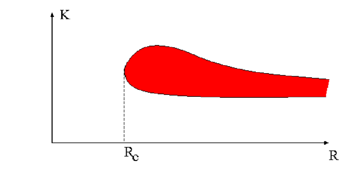
$$\text{spectrum}\,(\mathcal{A}) = \bigcup_{k_x,k_z}\text{spectrum}\left(\hat{\mathcal{A}}(k_x,k_z)\right)$$

Tollmien-Schlichting Instability

Poiseuille flow at $R = 6000$, $k_x = 1$, $k_z = 0$



Typical stability regions in K, R space: (for **Poiseuille** and **Blasius** boundary layer flows)



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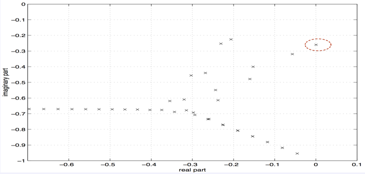

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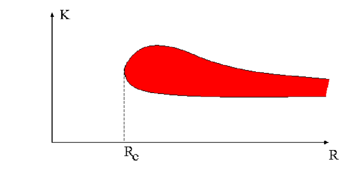
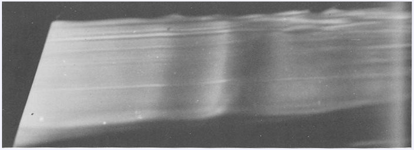


Unstable eigenvalue corresponds to a slowly growing traveling wave:

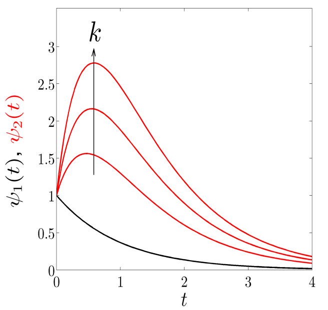
the *Tollmien-Schlichting* wave

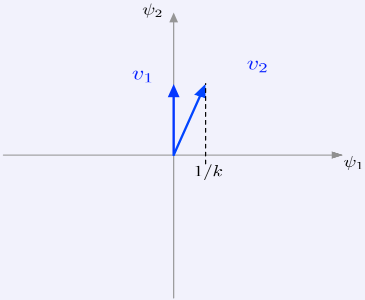
First seen in experiments by Skramstad & Schubauer, 1940



A Toy Example

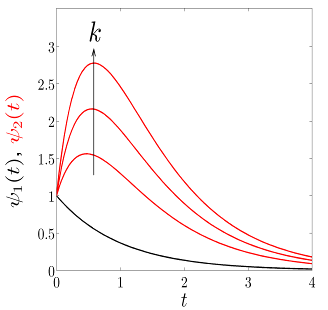
$$\begin{bmatrix} \dot{\psi}_1 \\ \dot{\psi}_2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ k & -2 \end{bmatrix} \begin{bmatrix} \psi_1 \\ \psi_2 \end{bmatrix}$$

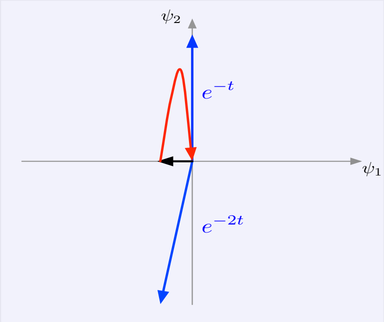




A Toy Example

$$\begin{bmatrix} \dot{\psi}_1 \\ \dot{\psi}_2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ {\color{red}k} & -2 \end{bmatrix} \begin{bmatrix} \psi_1 \\ \psi_2 \end{bmatrix}$$





Mathematical Modeling of Transition: Adding Signal Uncertainty

- Decompose the fields as

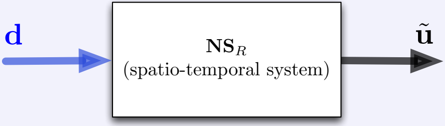
$$\mathbf{u} \;=\; \underset{\substack{\uparrow \\ \text{laminar}}}{\bar{\mathbf{u}}_R} \;+\; \underset{\substack{\uparrow \\ \text{fluctuations}}}{\tilde{\mathbf{u}}}$$

- Fluctuation dynamics:

In *linear* hydrodynamic stability, $-\nabla_{\tilde{u}}\tilde{u}$ is ignored

$$\begin{aligned} \partial_t \tilde{\mathbf{u}} &= -\nabla_{\bar{\mathbf{u}}_R}\tilde{\mathbf{u}} - \nabla_{\tilde{\mathbf{u}}}\bar{\mathbf{u}}_R - \text{grad}\,\tilde{p} + \tfrac{1}{R}\Delta\tilde{\mathbf{u}} - \nabla_{\tilde{u}}\tilde{u} + \mathbf{d} \\ 0 &= \text{div}\,\tilde{\mathbf{u}} \end{aligned}$$

  - a time-varying *exogenous disturbance* field $\mathbf{d}$



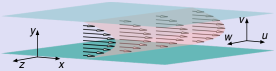
Input-Output view of the Linearized NS Equations

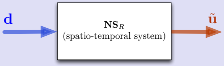
Jovanovic, BB, '05 JFM

Input-Output Analysis of the Linearized NS Equations

$$\partial_t \begin{bmatrix}\Delta\tilde v\\ \tilde\omega\end{bmatrix} = \begin{bmatrix} U''\partial_x - U\Delta\partial_x + \frac{1}{R}\Delta^2 & 0 \\ -U'\partial_z & -U\partial_x + \frac{1}{R}\Delta\end{bmatrix}\begin{bmatrix}\tilde v\\ \tilde\omega\end{bmatrix} + \begin{bmatrix} -\partial_{xy} & \partial_x^2+\partial_z^2 & -\partial_{zy}\\ \partial_z & 0 & -\partial_x\end{bmatrix}\begin{bmatrix}d_x\\ d_y\\ d_z\end{bmatrix}$$

$$\begin{bmatrix}\tilde u\\ \tilde v\\ \tilde w\end{bmatrix} = \left(\partial_x^2+\partial_z^2\right)^{-1}\begin{bmatrix}\partial_{xy} & -\partial_z\\ \partial_x^2+\partial_z^2 & 0\\ \partial_{zy} & \partial_x\end{bmatrix}\begin{bmatrix}\tilde v\\ \tilde\omega\end{bmatrix}$$





$$\begin{aligned}\partial_t\Psi &= \mathcal{A}\,\Psi + \mathcal{B}\,\mathbf{d}\\ \tilde{\mathbf{u}} &= \mathcal{C}\,\Psi\end{aligned}$$

Input-Output Analysis of the Linearized NS Equations

$$\partial_t \begin{bmatrix} \Delta \tilde{v} \\ \tilde{\omega} \end{bmatrix} = \begin{bmatrix} U''\partial_x - U\Delta\partial_x + \frac{1}{R}\Delta^2 & 0 \\ -U'\partial_z & -U\partial_x + \frac{1}{R}\Delta \end{bmatrix} \begin{bmatrix} \tilde{v} \\ \tilde{\omega} \end{bmatrix} + \begin{bmatrix} -\partial_{xy} & \partial_x^2 + \partial_z^2 & -\partial_{zy} \\ \partial_z & 0 & -\partial_x \end{bmatrix} \begin{bmatrix} d_x \\ d_y \\ d_z \end{bmatrix}$$

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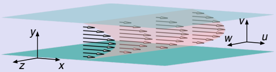

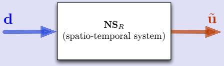

$$\begin{aligned} \partial_t \Psi &= \mathcal{A}\,\Psi + \mathcal{B}\,\mathbf{d} \\ \tilde{\mathbf{u}} &= \mathcal{C}\,\Psi \end{aligned}$$

- eigs $(\mathcal{A})$: determine stability
- System norm $\mathbf{d} \longrightarrow \tilde{\mathbf{u}}$: determines response to disturbances

Input-Output Analysis of the Linearized NS Equations

$$\partial_t \begin{bmatrix} \Delta\tilde{v} \\ \tilde{\omega} \end{bmatrix} = \begin{bmatrix} U''\partial_x - U\Delta\partial_x + \frac{1}{R}\Delta^2 & 0 \\ -U'\partial_z & -U\partial_x + \frac{1}{R}\Delta \end{bmatrix} \begin{bmatrix} \tilde{v} \\ \tilde{\omega} \end{bmatrix} + \begin{bmatrix} -\partial_{xy} & \partial_x^2 + \partial_z^2 & -\partial_{zy} \\ \partial_z & 0 & -\partial_x \end{bmatrix} \begin{bmatrix} d_x \\ d_y \\ d_z \end{bmatrix}$$

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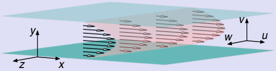

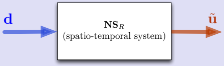

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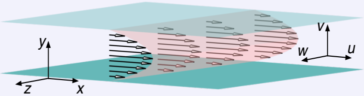
Surprises:

- Even when $\mathcal{A}$ is stable — the gain $\mathbf{d} \longrightarrow \tilde{\mathbf{u}}$ can be very large
- Input-output resonances — very different from least-damped modes of $\mathcal{A}$

Translation invariance in x & z implies



- Impulse Response

$$\tilde{\mathbf{u}}(t,x,y,z) = \int G(t-\tau, x-\xi, y, y', z-\zeta)\, \mathbf{d}(\tau,\xi,y',\zeta)\, d\tau d\xi dy' d\zeta$$

$$\tilde{\mathbf{u}}(t,x,.,z) = \int \mathcal{G}(t-\tau, x-\xi, z-\zeta)\, \mathbf{d}(\tau,\xi,.,\zeta)\, d\tau d\xi d\zeta$$

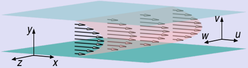
$\mathcal{G}(t,x,z)$: Operator-valued impulse response function

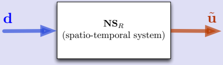
- Frequency Response

$$\tilde{\mathbf{u}}(\omega,k_x,k_z) = \mathcal{G}(\omega,k_x,k_z)\, \mathbf{d}(\omega,k_x,k_z)$$

$\mathcal{G}(\omega,k_x,k_z)$: Operator-valued frequency response. Packs lots of information!

Modal vs. Input-Output Analysis





- $$\begin{aligned} \partial_t \Psi &= \mathcal{A}\,\Psi + \mathcal{B}\,\mathbf{d} \\ \tilde{\mathbf{u}} &= \mathcal{C}\,\Psi \end{aligned}$$
- IR: $\mathcal{G}(t, x, z)$
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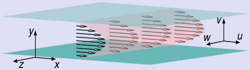

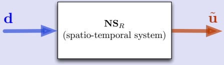

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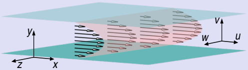
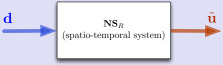
Modal Analysis: Look for unstable eigs of $\mathcal{A}$

Flow type	Classical linear theory R_c	Experimental R_c
Channel Flow	5772	$\approx$ 1,000-2,000
Plane Couette	∞	$\approx$ 350
Pipe Flow	∞	$\approx$ 2,200-100,000

Modal vs. Input-Output Analysis





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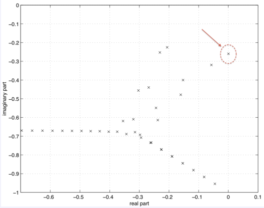
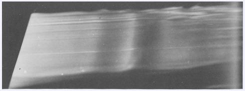
Modal Analysis: Look for unstable eigs of $\mathcal{A}$

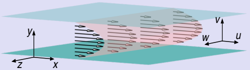
- Channel Flow @ $R = 6000$, $k_x = 1$, $k_z = 0$:

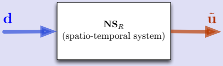


- Flow structure of corresponding eigenfunction: Tollmein-Schlichting (TS) waves



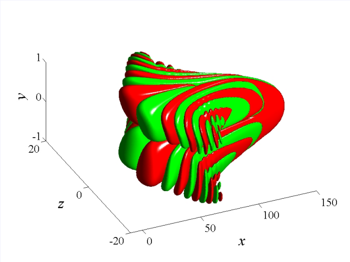
Modal vs. Input-Output Analysis

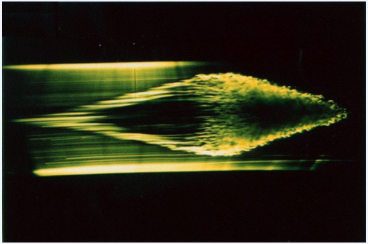




- $$\begin{aligned} \partial_t \Psi &= \mathcal{A}\,\Psi + \mathcal{B}\,\mathbf{d} \\ \tilde{\mathbf{u}} &= \mathcal{C}\,\Psi \end{aligned}$$
- IR: $G(t, x, y, -1, z)$
- FR: $\mathcal{G}(\omega, k_x, k_z)$

Impulse Response Analysis: Channel Flow @ $R = 2000$



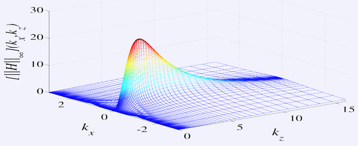


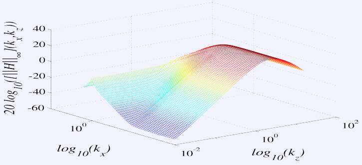
similar to "turbulent spots"

Spatio-temporal Frequency Response

$\mathcal{G}(\omega, k_x, k_z)$ is a *large* object!

one aggregation method: $\sup_\omega \sigma_{\max}\left(\mathcal{G}(\omega, k_x, k_z)\right)$





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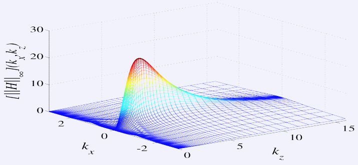
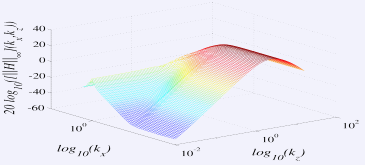
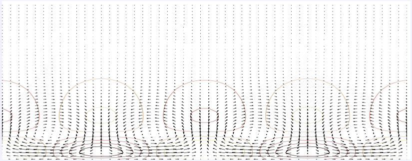
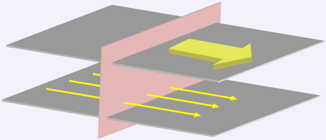




What do the corresponding flow structures look like?





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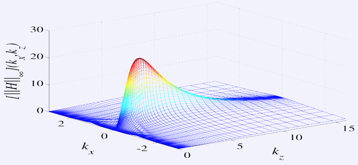

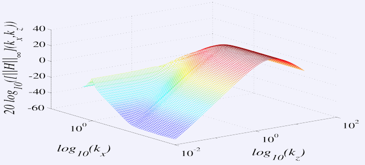

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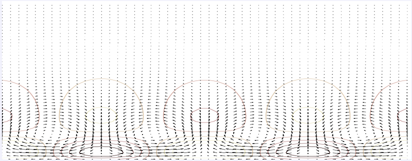

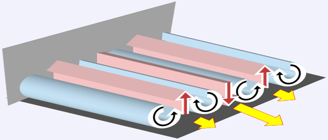

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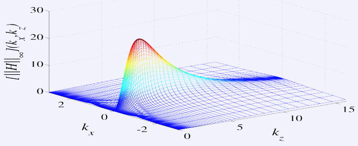
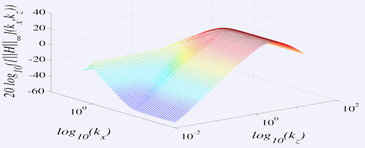
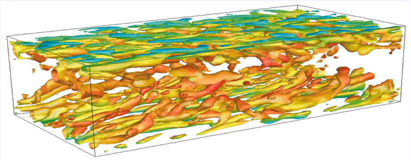
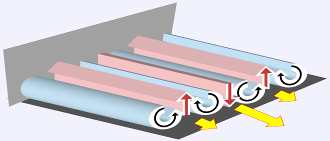




What do the corresponding flow structures look like?

closer (than TS waves) to structures seen in turbulent boundary layers



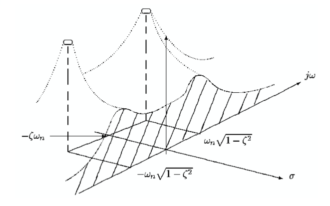


Modal vs. Input-Output Response

Correspondence: poles of a transfer function $\longleftrightarrow$ frequency response

Typically: underdamped poles $\longleftrightarrow$ frequency response peaks

cf. The "rubber sheet analogy":



Modal vs. Input-Output Response

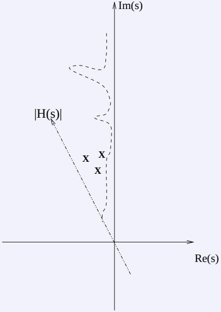
Correspondence: poles of a transfer function $\longleftrightarrow$ frequency response

However: no connection necessary between pole locations and FR peaks

Theorem: *Let $z_1, \ldots, z_n$ be any locations in the left half of the complex plane.*
Any stable frequency response function in $\mathcal{H}^2$ can be arbitrarily closely approximated by a transfer function of the following form:

$$H(s) = \sum_{i=1}^{N_1} \frac{\alpha_{1,i}}{(s-z_1)^i} + \cdots + \sum_{i=1}^{N_n} \frac{\alpha_{n,i}}{(s-z_n)^i}$$

by choosing any of the N_k's large enough



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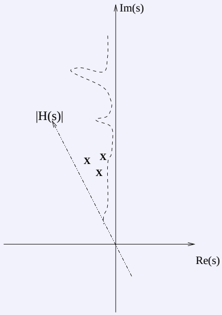
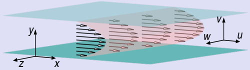
by choosing any of the N_k*'s large enough*

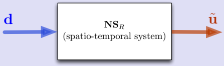


For large-scale systems: IO behavior not predictable from modal behavior

Implications for Turbulence

For large-scale systems: IO behavior not predictable from modal behavior





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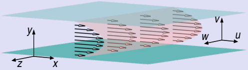

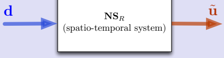

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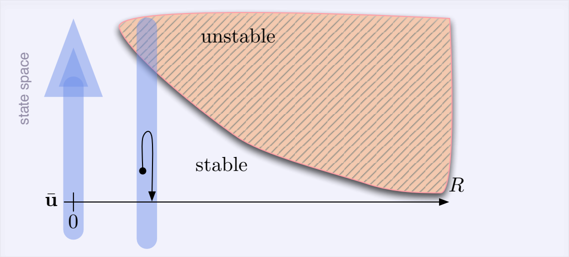
- "modal behavior": Stability due to i.c. condition uncertainty
- "IO behavior": behavior in the presence of ambient uncertainty
  - forcing terms from wall roughness and/or vibrations
  - Free-stream disturbances in boundary layers
  - Thermal (Langevin) forces
  - uncertain dynamics

The Nature of Turbulence

- Fluid flows are described by deterministic equations
- OLD QUESTION: why do fluid flows "look random" at high R?

The Nature of Turbulence

- **A common view of turbulence**



The Nature of Turbulence

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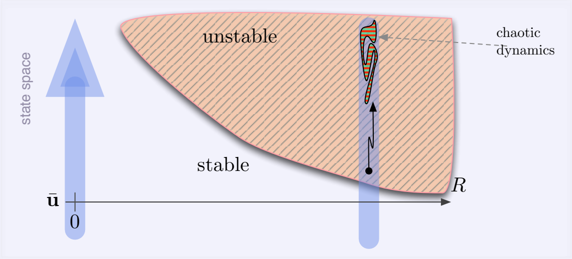

The Nature of Turbulence

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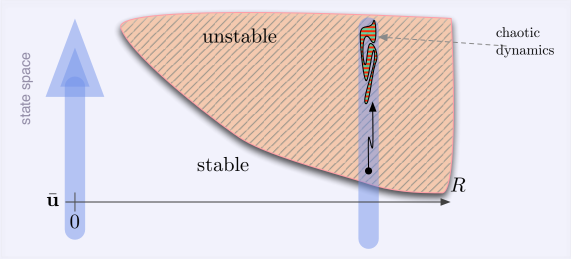

- **Intuitive reasoning:**
  Complex, “statistical looking” behavior $\longleftrightarrow$ chaotic dynamics

The Nature of Turbulence

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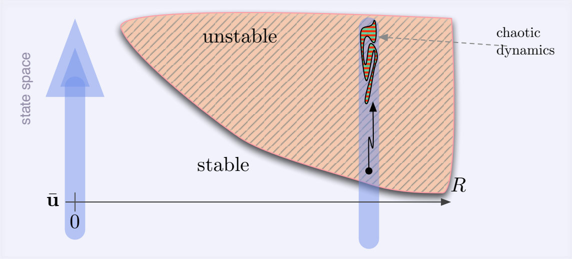
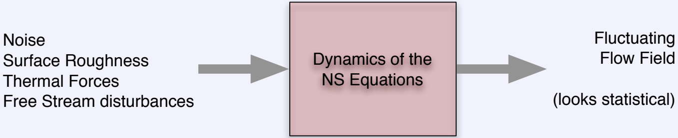


- **Intuitive reasoning:**
  Complex, "statistical looking" behavior $\longleftrightarrow$ chaotic dynamics
- Assumes NS eqs. with perfect BC, no disturbances or uncertainty
  (i.e. a a closed system)

The Nature of Turbulence

An Alternate Possibility

- A driven (open) system



The NS equations act as an *amplifier of ambient uncertainty* at high R

- Qualitatively similar to