All fluid flows *transition* (as $0 \xrightarrow{R} \infty$) from laminar to turbulent flows

- **Bluff bodies**
  - dominant phenomenon: *separation*

- **Streamlined bodies**
  - dominant phenomenon: *friction with walls*

*wall-bounded shear flows*
Boundary Layer Turbulence

Flow direction

boundary layer turbulence

side view

top view

Transition & Turbulence in Boundary Layer and Channel Flows
Boundary Layer Turbulence

boundary layer turbulence

Laminar boundary layer

Turbulent boundary layer

skin-friction drag: laminar vs. turbulent

- Transition & Turbulence in Boundary Layer and Channel Flows
- Technologically Important: Skin-Friction Drag
Control of Boundary Layer Turbulence

in nature: passive control

active control with sensor/actuator arrays

corrugated skin

compliant skin

Intuition: must have ability to actuate at spatial scale comparable to flow structures

Caveat: Plant's dynamics are not well understood:
- obstacles
- not only device technology
- also: dynamical modeling and control design
Control of Boundary Layer Turbulence

in nature:  *passive control*

*active control* with sensor/actuator arrays

- corrugated skin
- compliant skin

Intuition: must have ability to actuate at spatial scale comparable to flow structures

\[
\text{spatial-bandwidth of controller} \geq \text{plant’s bandwidth}
\]
Control of Boundary Layer Turbulence

- **passive control**

- **active control** with sensor/actuator arrays

- **corrugated skin**
- **compliant skin**

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  \[ \text{spatial-bandwidth of controller} \geq \text{plant's bandwidth} \]

- **Caveat:** Plant's dynamics are not well understood :(
  
  obstacles \{ not only device technology also: dynamical modeling and control design \}
The Navier-Stokes (NS) equations:

\[ \partial_t u = -\nabla u \cdot u - \text{grad} \ p + \frac{1}{R} \Delta u \]
\[ 0 = \text{div} \ u \]

- **Hydrodynamic Stability:** view NS as a dynamical system
- **laminar flow** \( \bar{u}_R \) := a stationary solution of the NS equations (an equilibrium)
Mathematical Modeling of Transition: Hydrodynamic Stability

The Navier-Stokes (NS) equations:

\[
\frac{\partial u}{\partial t} = -\nabla u - \text{grad} \, p + \frac{1}{R} \Delta u \\
0 = \text{div} \, u
\]

- Hydrodynamic Stability:
  - view NS as a dynamical system

- *laminar flow* \( \bar{u}_R \) := a stationary solution of the NS equations (an equilibrium)

**laminar flow** \( \bar{u}_R \) stable \( \iff \) 

i.c. \( u(0) \neq \bar{u}_R \),

\( u(t) \xrightarrow{t \to \infty} \bar{u}_R \)

- typically done with dynamics linearized about \( \bar{u}_R \)
- various methods to track further “non-linear behavior”
Mathematical Modeling of Transition: Hydrodynamic Stability

The Navier-Stokes (NS) equations:

\[
\partial_t \mathbf{u} = -\nabla \cdot \mathbf{u} - \text{grad } p + \frac{1}{R} \Delta \mathbf{u} \\
0 = \text{div } \mathbf{u}
\]

- Hydrodynamic Stability:
  - view NS as a dynamical system
- A very successful *(phenomenologically predictive)* approach for many decades
Mathematical Modeling of Transition: Hydrodynamic Stability

The Navier-Stokes (NS) equations:

\[ \partial_t \mathbf{u} = -\nabla \cdot \mathbf{u} - \text{grad} \, p + \frac{1}{R} \Delta \mathbf{u} \]

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- **Hydrodynamic Stability:** view NS as a dynamical system
- **however:** problematic for wall-bounded shear flows

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Mathematical Modeling of Transition: Hydrodynamic Stability

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- was widely believed: *this theory fails because it is linear and “nonlinear effects” are important even for infinitesimal i.c.*
- however, since 90’s: story is actually more interesting than that

Nonmodal Stability Theory, Schmid, ARFM ’07
Decompose the fields as

\[ u = \bar{u}_R + \tilde{u} \]

\[ \uparrow \quad \text{laminar fluctuations} \]

Fluctuation dynamics:

In linear hydrodynamic stability, \(-\nabla \tilde{u} \tilde{u}\) is ignored

\[
\partial_t \tilde{u} = -\nabla \bar{u}_R \tilde{u} - \nabla \tilde{u} \bar{u}_R - \text{grad} \tilde{p} + \frac{1}{R} \Delta \tilde{u} - \nabla \tilde{u} \tilde{u} \\
0 = \text{div} \tilde{u}
\]
Mathematical Modeling of Transition: Linearized Stability

- Decompose the fields as
  \[ u = \bar{u}_R + \tilde{u} \]
  \[ \uparrow \quad \text{laminar fluctuations} \quad \uparrow \]

- Fluctuation dynamics:
  In linear hydrodynamic stability, \(-\nabla_{\tilde{u}\tilde{u}}\) is ignored
  \[
  \partial_t \tilde{u} = -\nabla_{\bar{u}_R} \tilde{u} - \nabla \tilde{u} \bar{u}_R - \text{grad} \hat{p} + \frac{1}{R} \Delta \tilde{u} - \nabla \tilde{u} \tilde{u} \\
  0 = \text{div} \tilde{u}
  \]

- Linearization in “Evolution Form”
  \[
  \partial_t \begin{bmatrix} \Delta \tilde{v} \\ \tilde{\omega} \end{bmatrix} = \begin{bmatrix} U''' \partial_x - U \Delta \partial_x + \frac{1}{R} \Delta^2 & 0 \\
  -U' \partial_z & -U \partial_x + \frac{1}{R} \Delta \end{bmatrix} \begin{bmatrix} \tilde{v} \\ \tilde{\omega} \end{bmatrix} \\
  \tilde{v} := \text{wall-normal velocity} \\
  \tilde{\omega} := \text{wall-normal vorticity}
  \]
  \[=:\begin{bmatrix} \mathcal{L} & 0 \\ -\mathcal{C} & S \end{bmatrix} \begin{bmatrix} \tilde{v} \\ \tilde{\omega} \end{bmatrix} =: A \begin{bmatrix} \tilde{v} \\ \tilde{\omega} \end{bmatrix} \]
Decompose the fields as
\[ u = \bar{u}_R + \tilde{u} \]
\[ \uparrow \quad \uparrow \]
laminar fluctuations

Fluctuation dynamics:
In linear hydrodynamic stability, \(-\nabla \tilde{u} \tilde{u}\) is ignored

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\begin{align*}
\partial_t \tilde{u} &= -\nabla \bar{u}_R \tilde{u} - \nabla \tilde{u} \bar{u}_R - \text{grad} \hat{p} + \frac{1}{R} \Delta \tilde{u} - \nabla \tilde{u} \tilde{u} \\
0 &= \text{div} \tilde{u}
\end{align*}
\]

Linearization in “Evolution Form”

\[
\begin{align*}
\partial_t \begin{bmatrix} \Delta \tilde{v} \\ \tilde{\omega} \end{bmatrix} &= \begin{bmatrix} U''' \partial_x - U \Delta \partial_x + \frac{1}{R} \Delta^2 & 0 \\
-U' \partial_z & -U \partial_x + \frac{1}{R} \Delta \end{bmatrix} \begin{bmatrix} \tilde{v} \\ \tilde{\omega} \end{bmatrix} \\
&=: \begin{bmatrix} \mathcal{L} & 0 \\ \mathcal{C} & S \end{bmatrix} \begin{bmatrix} \tilde{v} \\ \tilde{\omega} \end{bmatrix} =: \mathcal{A} \begin{bmatrix} \tilde{v} \\ \tilde{\omega} \end{bmatrix}
\end{align*}
\]

\[ \tilde{v} := \text{wall-normal velocity} \]
\[ \tilde{\omega} := \text{wall-normal vorticity} \]

Classical Linear Hydrodynamic Stability:

Transition \[\leftrightarrow\] Instability \[\leftrightarrow\] \(\mathcal{A}\) has spectrum in right half plane

The existence of “exponentially growing normal modes” (of \(e^{t\mathcal{A}}\)
For parallel channel flows

\( \mathcal{A} \) is \textit{translation invariant} in \( x, z \), \( \Rightarrow \) Fourier transform in \( x \) and \( z \):

\[
\frac{\partial}{\partial t} \begin{bmatrix} \hat{v} \\ \hat{\omega} \end{bmatrix} = \begin{bmatrix} i k_x \Delta^{-1} U''' - i k_x \Delta^{-1} U \Delta + \frac{1}{R} \Delta^{-1} \Delta^2 & 0 \\ -i k_z U' & -i k_x U + \frac{1}{R} \Delta \end{bmatrix} \begin{bmatrix} \hat{v} \\ \hat{\omega} \end{bmatrix}
\]

\( k_x, k_z \): spatial frequencies in \( x, z \) directions (wave-numbers).

\[
\frac{\partial}{\partial t} \begin{bmatrix} \hat{v}(t, k_x, .., k_z) \\ \hat{\omega}(t, k_x, .., k_z) \end{bmatrix} = \hat{A}(k_x, k_z) \begin{bmatrix} \hat{v}(t, k_x, .., k_z) \\ \hat{\omega}(t, k_x, .., k_z) \end{bmatrix}
\]

Essentially:

\[
\text{spectrum } (\mathcal{A}) = \bigcup_{k_x, k_z} \text{spectra } \left( \hat{A}(k_x, k_z) \right)
\]
Poiseuille flow at $R = 6000$, $k_x = 1$, $k_z = 0$

Typical stability regions in $K$, $R$ space: (for Poiseuille and Blasius boundary layer flows)
Tollmien-Schlichting Instability

Poiseuille flow at $R = 6000$, $k_x = 1$, $k_z = 0$

Typical stability regions in $K$, $R$ space: (for Poiseuille and Blasius boundary layer flows)

Unstable eigenvalue corresponds to a slowly growing traveling wave:

the Tollmien-Schlichting wave

First seen in experiments by Skramstad & Schubauer, 1940
A Toy Example

\[
\begin{bmatrix}
\dot{\psi}_1 \\
\dot{\psi}_2
\end{bmatrix} = 
\begin{bmatrix}
-1 & 0 \\
k & -2
\end{bmatrix}
\begin{bmatrix}
\psi_1 \\
\psi_2
\end{bmatrix}
\]
A Toy Example

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↑

laminar fluctuations

Fluctuation dynamics:

In linear hydrodynamic stability, \(-\nabla \tilde{u} \tilde{u}\) is ignored

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\begin{align*}
\partial_t \tilde{u} &= -\nabla \bar{u}_R \tilde{u} - \nabla \tilde{u} \bar{u}_R - \text{grad} \tilde{p} + \frac{1}{R} \Delta \tilde{u} - \nabla \bar{u} \tilde{u} + d \\
0 &= \text{div} \tilde{u}
\end{align*}
\]

- a time-varying *exogenous disturbance* field \(d\)

\[ \text{NS}_R \] (spatio-temporal system)

Input-Output view of the Linearized NS Equations

*Jovanovic, BB, '05 JFM*
Input-Output Analysis of the Linearized NS Equations

\[
\partial_t \begin{bmatrix} \Delta \tilde{v} \\ \tilde{\omega} \end{bmatrix} = \begin{bmatrix} U'' \partial_x - U \Delta \partial_x + \frac{1}{R} \Delta^2 & 0 \\ -U' \partial_z & -U \partial_x + \frac{1}{R} \Delta \end{bmatrix} \begin{bmatrix} \tilde{v} \\ \tilde{\omega} \end{bmatrix} + \begin{bmatrix} -\partial_{xy} & \partial_x^2 & \partial_z^2 & -\partial_{zy} \\ \partial_x & 0 & -\partial_x \end{bmatrix} \begin{bmatrix} d_x \\ d_y \\ d_z \end{bmatrix}
\]

\[
\begin{bmatrix} \tilde{u} \\ \tilde{v} \\ \tilde{w} \end{bmatrix} = \left( \partial_x^2 + \partial_z^2 \right)^{-1} \begin{bmatrix} \partial_{xy} & -\partial_z \\ \partial_x^2 + \partial_z^2 & 0 \\ \partial_{zy} & \partial_x \end{bmatrix} \begin{bmatrix} \tilde{v} \\ \tilde{\omega} \end{bmatrix}
\]

\[
\partial_t \Psi = A \Psi + B d
\]

\[
\tilde{u} = C \Psi
\]
Input-Output Analysis of the Linearized NS Equations

\[ \partial_t \begin{bmatrix} \Delta \tilde{v} \\ \tilde{\omega} \end{bmatrix} = \begin{bmatrix} U'' \partial_x & - U \Delta \partial_x + \frac{1}{R} \Delta^2 \\ -U' \partial_z & -U \partial_x + \frac{1}{R} \Delta \end{bmatrix} \begin{bmatrix} \tilde{v} \\ \tilde{\omega} \end{bmatrix} + \begin{bmatrix} -\partial_{xy} & \partial_y^2 + \partial_z^2 & -\partial_{zy} \\ \partial_x \partial_y & 0 & -\partial_x \end{bmatrix} \begin{bmatrix} \tilde{v} \\ \tilde{\omega} \end{bmatrix} \]

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\[ \partial_t \Psi = A \Psi + B \mathbf{d} \]

\[ \tilde{\mathbf{u}} = C \Psi \]

- eigs (\(A\)): determine stability
- System norm \(\mathbf{d} \rightarrow \tilde{\mathbf{u}}\): determines response to disturbances
Input-Output Analysis of the Linearized NS Equations

\[
\partial_t \begin{bmatrix} \Delta \tilde{v} \\ \tilde{\omega} \end{bmatrix} = \begin{bmatrix} U'' \partial_x - U \Delta \partial_x + \frac{1}{R} \Delta^2 & 0 \\ -U' \partial_z & -U \partial_x + \frac{1}{R} \Delta \end{bmatrix} \begin{bmatrix} \tilde{v} \\ \tilde{\omega} \end{bmatrix} + \begin{bmatrix} -\partial_{xy} & \partial_x^2 & \partial_z^2 & -\partial_{zy} \\ \partial_z & 0 & -\partial_x \end{bmatrix} \begin{bmatrix} d_x \\ d_y \\ d_z \end{bmatrix}
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\begin{bmatrix} \tilde{u} \\ \tilde{v} \\ \tilde{w} \end{bmatrix} = \left( \partial^2_x + \partial^2_z \right)^{-1} \begin{bmatrix} \partial_{xy} & -\partial_z \\ \partial_x^2 + \partial_z^2 & 0 \\ \partial_{zy} & \partial_x \end{bmatrix} \begin{bmatrix} \tilde{v} \\ \tilde{\omega} \end{bmatrix}
\]

\[
\begin{aligned}
\partial_t \Psi &= A \Psi + B d \\
\tilde{u} &= C \Psi
\end{aligned}
\]

Surprises:
- Even when \( A \) is stable, the gain \( d \rightarrow \tilde{u} \) can be very large.
- Input-output resonances very different from least-damped modes of \( A \).
Spatio-temporal Impulse and Frequency Responses

Translation invariance in $x$ & $z$ implies

- **Impulse Response**

  \[
  \tilde{u}(t, x, y, z) = \int G(t - \tau, x - \xi, y, y', z - \zeta) \, d\tau \, d\xi \, dy' \, d\zeta
  \]

  \[
  \tilde{u}(t, x, .., z) = \int G(t - \tau, x - \xi, .., z - \zeta) \, d\tau \, d\xi \, d\zeta
  \]

  \[G(t, x, z)\] : Operator-valued impulse response function

- **Frequency Response**

  \[
  \tilde{u}(\omega, k_x, k_z) = \mathcal{G}(\omega, k_x, k_z) \, d(\omega, k_x, k_z)
  \]

  \[\mathcal{G}(\omega, k_x, k_z)\] : Operator-valued frequency response. Packs lots of information!
Modal vs. Input-Output Analysis

\[ \partial_t \Psi = A \Psi + B \, d \]

\[ \tilde{\mathbf{u}} = C \, \Psi \]

- IR: \( G(t, x, z) \)
- FR: \( G(\omega, k_x, k_z) \)
Modal vs. Input-Output Analysis

\[
\frac{\partial t}{\partial t} \Psi = A \Psi + B \mathbf{d}
\]
\[
\tilde{\mathbf{u}} = C \Psi
\]

IR: \( \mathcal{G}(t, x, z) \)
FR: \( \mathcal{G}(\omega, k_x, k_z) \)

**Modal Analysis:** Look for unstable eigs of \( \mathcal{A} \)

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Modal vs. Input-Output Analysis

\[
\begin{align*}
\partial_t \Psi &= A \Psi + B \, \mathbf{d} \\
\tilde{\mathbf{u}} &= C \, \Psi \\
\text{IR: } &G(t, x, z) \\
\text{FR: } &G(\omega, k_x, k_z)
\end{align*}
\]

**Modal Analysis:** Look for unstable eigs of \( A \)

- Channel Flow @ \( R = 6000, k_x = 1, k_z = 0 \):

- Flow structure of corresponding eigenfunction:
  Tollmein-Schlichting (TS) waves
Modal vs. Input-Output Analysis

\[ \partial_t \Psi = A \Psi + B \mathbf{d} \]
\[ \mathbf{\tilde{u}} = C \Psi \]

IR: \( G(t, x, y, -1, z) \)
FR: \( G(\omega, k_x, k_z) \)

Impulse Response Analysis: Channel Flow @ \( R = 2000 \)

similar to “turbulent spots”
$\mathcal{G}(\omega, k_x, k_z)$ is a large object!

one aggregation method: $\sup_{\omega} \sigma_{\text{max}} \left( \mathcal{G}(\omega, k_x, k_z) \right)$
Spatio-temporal Frequency Response

\[ G(\omega, k_x, k_z) \text{ is a large object!} \]

one aggregation method: \( \sup_{\omega} \sigma_{\text{max}} \left( G(\omega, k_x, k_z) \right) \)

What do the corresponding flow structures look like?
$G(\omega, k_x, k_z)$ is a large object!

one aggregation method: $\sup_{\omega} \sigma_{\text{max}} \left( G(\omega, k_x, k_z) \right)$

What do the corresponding flow structures look like?
Spatio-temporal Frequency Response

$G(\omega, k_x, k_z)$ is a large object!

one aggregation method: $\sup_{\omega} \sigma_{\text{max}} \left( G(\omega, k_x, k_z) \right)$

What do the corresponding flow structures look like?
closer (than TS waves) to structures seen in turbulent boundary layers
Modal vs. Input-Output Response

Correspondence: poles of a transfer function $\leftrightarrow$ frequency response

Typically: underdamped poles $\leftrightarrow$ frequency response peaks

cf. The “rubber sheet analogy”: 
Modal vs. Input-Output Response

Correspondence: poles of a transfer function \( \longleftrightarrow \) frequency response

However: no connection necessary between pole locations and FR peaks

**Theorem:** Let \( z_1, \ldots, z_n \) be any locations in the left half of the complex plane. Any stable frequency response function in \( \mathcal{H}^2 \) can be arbitrarily closely approximated by a transfer function of the following form:

\[
H(s) = \sum_{i=1}^{N_1} \frac{\alpha_{1,i}}{(s - z_1)^i} + \cdots + \sum_{i=1}^{N_n} \frac{\alpha_{n,i}}{(s - z_n)^i}
\]

by choosing any of the \( N_k \)'s large enough.
Modal vs. Input-Output Response

Correspondence: poles of a transfer function \( \leftrightarrow \) frequency response

However: no connection necessary between pole locations and FR peaks

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by choosing any of the \( N_k \)'s large enough

For large-scale systems: IO behavior not predictable from modal behavior
Implications for Turbulence

For large-scale systems: IO behavior not predictable from modal behavior

\[ \frac{\partial}{\partial t} \Psi = A \Psi + B \mathbf{d} \]

\[ \tilde{\mathbf{u}} = C \Psi \]

- IR: \( G(t, x, z) \)
- FR: \( G(\omega, k_x, k_z) \)

\( \text{NS}_R \) 
(spatio-temporal system)
Implications for Turbulence

For large-scale systems: IO behavior not predictable from modal behavior

- "modal behavior": Stability due to i.c. condition uncertainty
- "IO behavior": behavior in the presence of ambient uncertainty
  - forcing terms from wall roughness and/or vibrations
  - Free-stream disturbances in boundary layers
  - Thermal (Langevin) forces
  - uncertain dynamics

\[
\begin{align*}
\partial_t \Psi &= A \Psi + B d \\
\tilde{u} &= C \Psi
\end{align*}
\]

IR: \( G(t, x, z) \)
FR: \( G(\omega, k_x, k_z) \)
**The Nature of Turbulence**

- Fluid flows are described by deterministic equations
- **Old Question:** why do fluid flows “look random” at high $R$?
The Nature of Turbulence

- A common view of turbulence
The Nature of Turbulence

- A common view of turbulence
The Nature of Turbulence

- **A common view of turbulence**

  ![Diagram showing state space with regions labeled as stable, unstable, and chaotic dynamics.]

- **Intuitive reasoning:**
  Complex, “statistical looking” behavior $\longleftrightarrow$ chaotic dynamics
**A common view of turbulence**

Intuitive reasoning:
- Complex, “statistical looking” behavior \(\leftrightarrow\) chaotic dynamics
- Assumes NS eqs. with perfect BC, no disturbances or uncertainty (i.e. a closed system)
The Nature of Turbulence
An Alternate Possibility

- A driven (open) system

Noise
Surface Roughness
Thermal Forces
Free Stream disturbances

Dynamics of the NS Equations

Fluctuating Flow Field
(looks statistical)

The NS equations act as an *amplifier of ambient uncertainty* at high $R$

- Qualitatively similar to

Noise
Surface Roughness
Thermal Forces
Free Stream disturbances

Dynamics of the *Linearized* NS Equations

Fluctuating Flow Field
(looks statistical)