Hamilton-Jacobi ENO

The goal is to construct ϕ_x^- and ϕ_x^+ at each grid point x_i. We write $\phi_x = \frac{dQ}{dx} = \frac{dQ_1}{dx} + \frac{dQ_2}{dx} + \frac{dQ_3}{dx}$. We suppose that we have computed the divided difference tables D^1, D^2 and D^3.

Algorithm:

1. To find ϕ_x^-, start with $k = i - 1$, and to find ϕ_x^+, start with $k = i$.
2. Define $\frac{dQ_1}{dx}(x_i) = D^1_{k+1/2}\phi$.
3. (a) If $|D^2_k\phi| < |D^2_{k+1}\phi|$ set $c = D^2_k\phi$ and $k^* = k - 1$,
   (b) Else set $c = D^2_{k+1}\phi$ and $k^* = k$.
   (c) Define $\frac{dQ_2}{dx}(x_i) = c\,(2(i-k)-1)\,\triangle x$.
4. (a) If $|D^3_{k^*+1/2}\phi| < |D^3_{k^*+3/2}\phi|$ set $c^* = D^3_{k^*+1/2}\phi$,
   (b) Else set $c^* = D^3_{k^*+3/2}\phi$.
   (c) Define $\frac{dQ_3}{dx}(x_i) = c^*\left(3(i-k^*)^2 - 6(i-k^*) + 2\right)\triangle x^2$.

Notes: ϕ_y^-, ϕ_y^+, ϕ_z^- and ϕ_z^+ are obtained similarly.