Hamilton-Jacobi ENO

The goal is to construct $\phi_x^-$ and $\phi_x^+$ at each grid point $x_i$. We write $\phi_x = \frac{dQ_1}{dx} + \frac{dQ_2}{dx} + \frac{dQ_3}{dx}$. We suppose that we have computed the divided difference tables $D^1$, $D^2$ and $D^3$.

Algorithm:

1. To find $\phi_x^-$, start with $k = i - 1$, and to find $\phi_x^+$, start with $k = i$.

2. Define $\frac{dQ_1}{dx}(x_i) = D^1_{k+1/2}\phi$.

3. (a) If $|D^2_k\phi| < |D^2_{k+1}\phi|$ set $c = D^2_k\phi$ and $k^* = k - 1$,
   (b) Else set $c = D^2_{k+1}\phi$ and $k^* = k$.
   (c) Define $\frac{dQ_2}{dx}(x_i) = c(2(i-k)-1)\triangle x$.

4. (a) If $|D^3_{k^*+1/2}\phi| < |D^3_{k^*+3/2}\phi|$ set $c^* = D^3_{k^*+1/2}\phi$,
   (b) Else set $c^* = D^3_{k^*+3/2}\phi$.
   (c) Define $\frac{dQ_3}{dx}(x_i) = c^* (3(i-k^*)^2 - 6(i-k^*) + 2) \triangle x^2$.

Notes: $\phi_y^-$, $\phi_y^+$, $\phi_z^-$ and $\phi_z^+$ are obtained similarly.