Level Set Methods and Their Applications

The goal of this homework is to implement the 'Level Set Equation' in two spatial dimensions

$$\phi_t + \mathbf{u}_{ext} \cdot \nabla \phi = 0,$$

where the external velocity $\mathbf{u}_{ext}$ does not depend on ϕ. We will do this in two steps:

1 Using the ENO scheme, solve the one dimensional advection equation with a square initial profile (makes sure ENO works in 1D).

2 Using this discretization and a dimension-by-dimension framework, we will solve the two dimensional advection equation to follow a circular interface moving with $(\sqrt{2}/2, \sqrt{2}/2)$. This circular interface will be defined as the zero cross-section of a well-chosen level set function.

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Consider a domain $\Omega = [-1, 1]$ in one spatial dimension and the following linear advection equation:

$$\phi_t + u\phi_x = 0, \tag{1}$$

where the velocity $u = 1$ is constant. The initial profile for ϕ is:

$$\phi(x, 0) = \begin{cases} 0, & x < -1/3; \\ 1, & -1/3 \leq x \leq 1/3; \\ 0, & x > 1/3. \end{cases}$$

1) Using the ENO scheme[1], solve equation (1) with the initial profile given above to a final time $t_{final} = 1/3$. We will define ϕ outside the domain by linear extrapolation as described in class. Take a time step restriction of $\triangle t = .5\triangle x$ and the number of grid points to be 100.

Same question using the velocity $u = -1$.

What to turn in: On a single figure, plot the initial ϕ along with the two solutions at t_{final}.

2) Consider a domain $\Omega = [-1, 1] \times [-1, 1]$ in two spatial dimensions. Define ϕ initially by $\phi = \sqrt{x^2 + y^2} - .3$ (note that the zero level set defines a circle with origin $(0, 0)$ and radius .3). Evolve ϕ in order to translate the circle with velocity $\mathbf{u}_{ext} = (\sqrt{2}/2, \sqrt{2}/2)$ by solving:

$$\phi_t + \mathbf{u}_{ext} \cdot \nabla \phi = 0$$

We will use 100 grid points in each spatial dimension and we will define ϕ outside the domain by linear extrapolation as described in class. The time step restriction will be $\triangle t \, (1/\triangle x + 1/\triangle y) = .5$. You can choose $t_{final} = 1/3$.

What to turn in: On a single figure, plot the initial interface and the interface translated. To plot this, use for example the command 'contour' in Matlab (type 'help contour' at the Matlab prompt to get a tutorial).

[1] Use first, or second or third order accurate, whichever you can implement.