Level Set Methods and Their Applications

Homework 2

Consider the following conservation laws:

$$u_t + [f(u)]_x = 0,$$

and its approximation:

$$Q_i^{n+1} = Q_i^n - \frac{\triangle t}{\triangle x} \left(F_{i+1/2}^n - F_{i-1/2}^n \right),$$

where

$$F_{i-1/2}^n \approx \frac{1}{\triangle t} \int_{t^n}^{t^{n+1}} f(u(x_{i-1/2}, t))dt,$$

and

$$Q_i^n = \frac{1}{\triangle x} \int_{x_{i-1/2}}^{x_{i+1/2}} u(x, t^n)dx,$$

Suppose that $f'' < 0$. Write down an expression for $F_{i-1/2}^n$ and explain your choice by referring to the characteristic curves.