Level Set Methods and Their Applications

1) Consider a domain $\Omega = [-1, 1] \times [-1, 1]$ in two spatial dimensions. The goal is to define ϕ as a signed distance function in such a way its zero contour represents a square centered at the origin and with side's length 1.

Define ϕ initially by $\phi(x, y) = -1$ if (x, y) is inside the square and $\phi(x, y) = 1$ if (x, y) is outside the square. Then 'reinitialize' ϕ as a signed distance function by solving:

$$\phi_t + \text{Sign}(\phi)\,(|\nabla\phi| - 1) = 0$$

to steady state using the Godunov scheme described in class. Use 100 grid points in each spatial dimension.

What to turn in: A plot of the sign distance function. To plot this, use for example the command 'mesh" in Matlab (type 'help mesh' at the Matlab prompt for a tutorial).

2) Using the level set function ϕ initialized in 1), solve the evolution equation in the positive normal direction up to $t_{final} = .3$:

$$\phi_t + \mathbf{n} \cdot \nabla\phi = 0$$

What to turn in: A plot of the zero contour at 10 successive time steps. To plot this, use for example the command 'contour' in Matlab (type 'help contour' at the Matlab prompt for a tutorial). Use also the command 'hold on' to plot several contours on a single plot.

3) Using the level set function ϕ initialized in 1), solve the evolution equation in the positive normal direction up to $t_{final} = .3$:

$$\phi_t - \mathbf{n} \cdot \nabla\phi = 0$$

What to turn in: A plot of the zero contour at 10 successive time steps. To plot this, use for example the command 'contour' in Matlab (type 'help contour' at the Matlab prompt for a tutorial). Use also the command 'hold on' to plot several contours on a single plot.