Wet Chemical Etching in Semiconductor Fabrication

Physical problem A gap of width 2a and length L is to be etched in a flat plate. The remainder of the plate is covered with a protective (photoresist) layer. Since it is assumed that L is much larger than 2a, the problem can be considered as two dimensional.

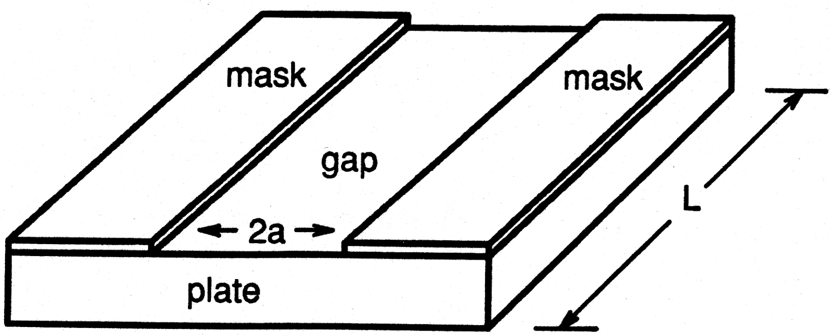


Figure 1: Physical Problem.

Assumptions:

there is no convection in the etching medium;
the etching process is isotropic;
the thickness of the photoresist layer is infinitely small;
only one component of the etching liquid determines the process.

Mathematical Model

The etching fluid $\Omega(t)$ is bounded by the outer boundary Γ_1; the photoresist layer $\Gamma_2(t)$, and the moving boundary S(t). $D\backslash\Omega(t)$ denotes part of the solid.

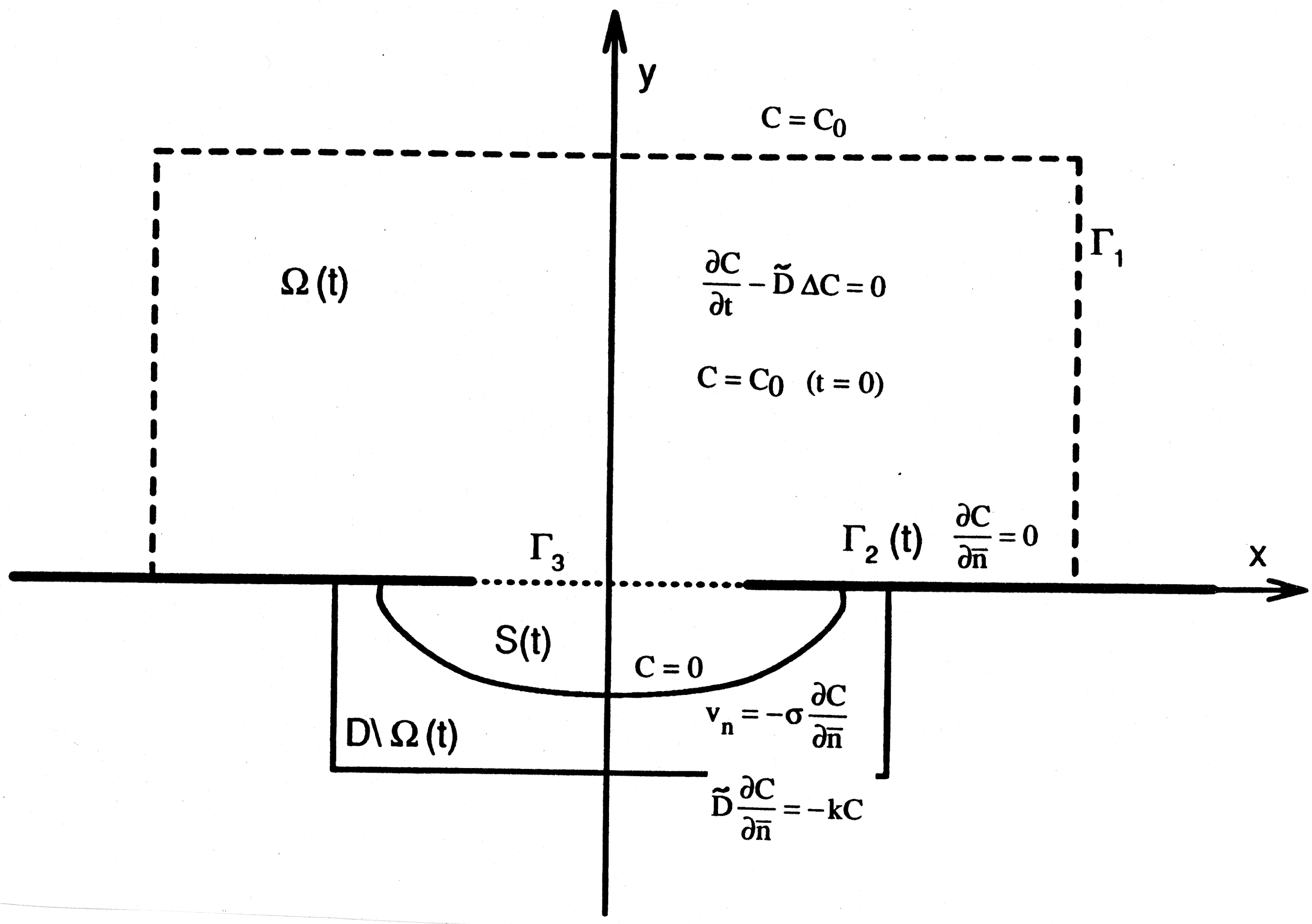
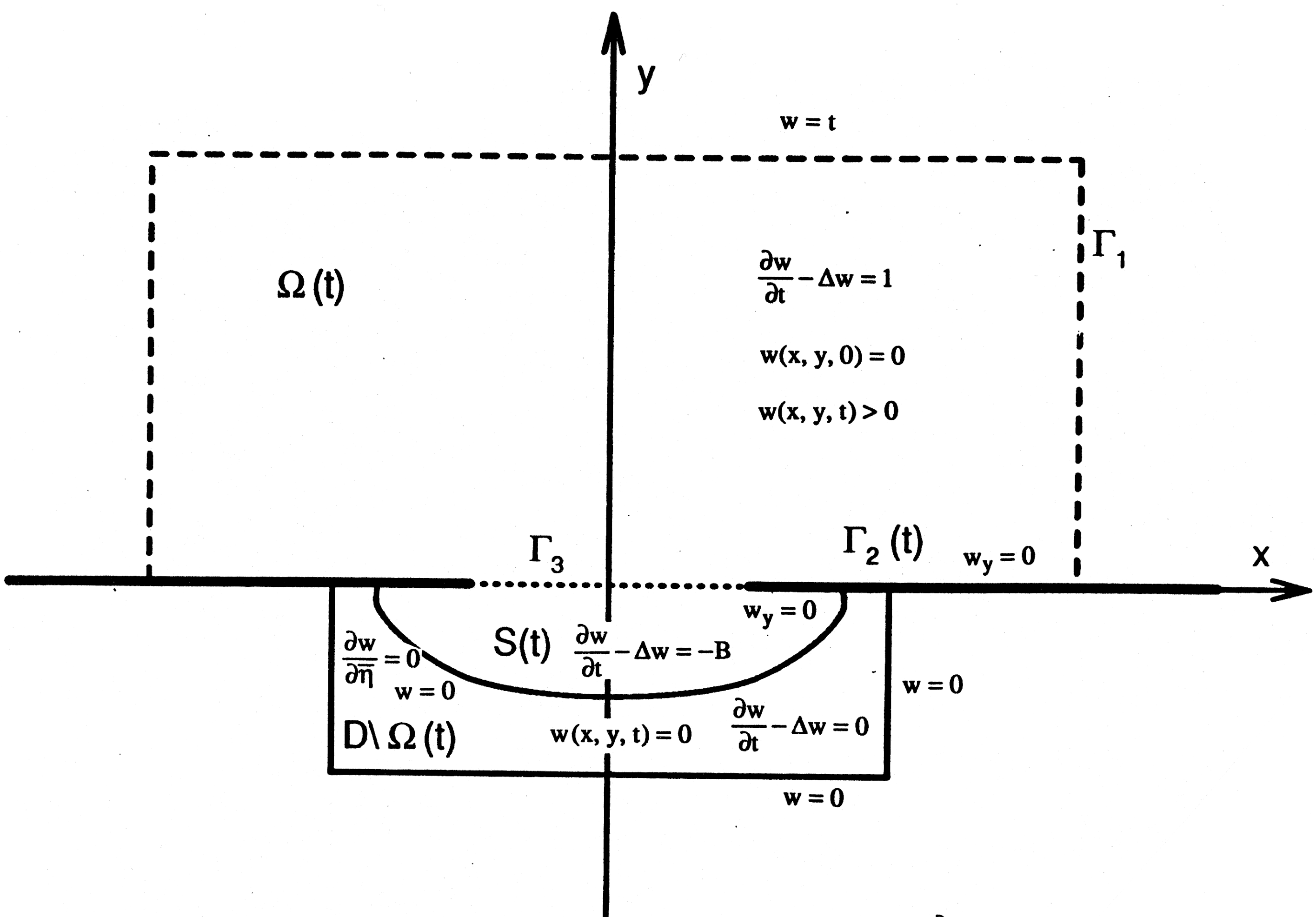


Figure 2: Side view of physical problem showing mathematical solution setup.

Fixed Domain Formulation



$$w(x,y,t) = \int_0^t \tilde{C}(x,y,\tau)d\tau \quad (x,y) \in D, \quad t \in (0,T)$$

$$\tilde{C} = \frac{\partial w}{\partial t}$$

Figure 3: Fixed domain mathematical formulation.

Numerical Algorithm

The basic numerical algorithm is:

$$(w_1)_{i,j,k}^{(n+1/2)} = \left(\frac{1}{\Delta t} + \frac{2}{(\Delta x)^2} + \frac{2}{(\Delta y)^2}\right)^{-1} \left\{C_0 + \frac{1}{\Delta t}(w_1)_{i,j,k-1} + \frac{1}{(\Delta x)^2}\left[(w_1)_{i-1,j,k}^{(n+1)} + (w_1)_{i+1,j,k}^{(n)}\right] + \frac{1}{(\Delta y)^2}\left[(w_1)_{i,j-1,k}^{(n+1)} + (w_1)_{i,j+1,k}^{(n)}\right]\right\}$$

with

$$(w_1)_{i,j,k}^{(n+1)} = (w_1)_{i,j,k}^{(n)} + \theta[(w_1)_{i,j,k}^{(n+1/2)} - (w_1)_{i,j,k}^{(n)}]$$

in $\Omega(0)$ and

$$(w_2)_{i,j,k}^{(n+1/2)} = \left(\frac{1}{\Delta t} + \frac{32}{(\Delta x)^2} + \frac{32}{(\Delta y)^2}\right)^{-1} \left\{-B + \frac{1}{\Delta t}(w_2)_{i,j,k-1} + \frac{16}{(\Delta x)^2}\left[(w_2)_{i-1,j,k}^{(n+1)} + (w_2)_{i+1,j,k}^{(n)}\right] + \frac{16}{(\Delta y)^2}\left[(w_2)_{i,j-1,k}^{(n+1)} + (w_2)_{i,j+1,k}^{(n)}\right]\right\}$$

with

$$(w_2)_{i,j,k}^{(n+1)} = \max\{0, (w_2)_{i,j,k}^{(n)} + \theta[(w_2)_{i,j,k}^{(n+1/2)} - (w_2)_{i,j,k}^{(n)}]\}$$

in D_1

Domain Decomposition

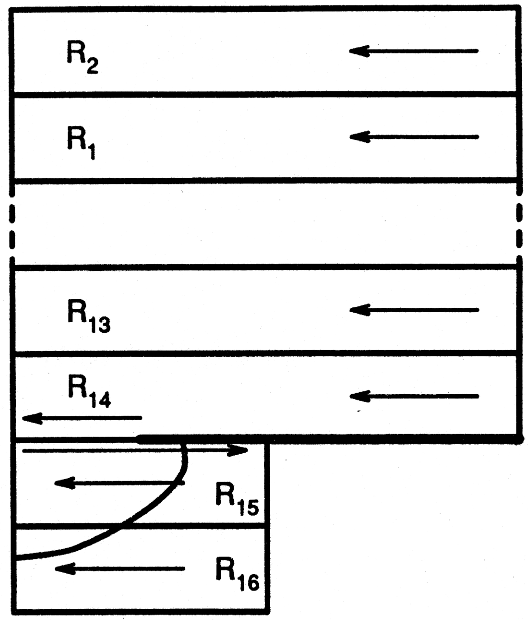


Figure 4: Domain decomposition of mathematical problem into sixteen subregions showing the flow of computations in each.

Load Balancing

Table: Load Balancing Information for Example

Nodes	2	4	8	16	32	64
bottom nodes	1	1	1	2	4	8
bottom points	12880	12880	12880	6440	3220	1610
top nodes	1	3	7	14	28	56
top points	89880	$\leq$ 30174	12840	6420	3210	1605
diff points	77000	17294	40	20	10	5

Speedup

$$S_p = \frac{T_1}{T_p}$$

where T_p is the time required to execute the algorithm using p processors and T_1 is the time required to execute the same program on a single processor.

Efficiency

$$e_p = \frac{S_p}{p}$$

Results:

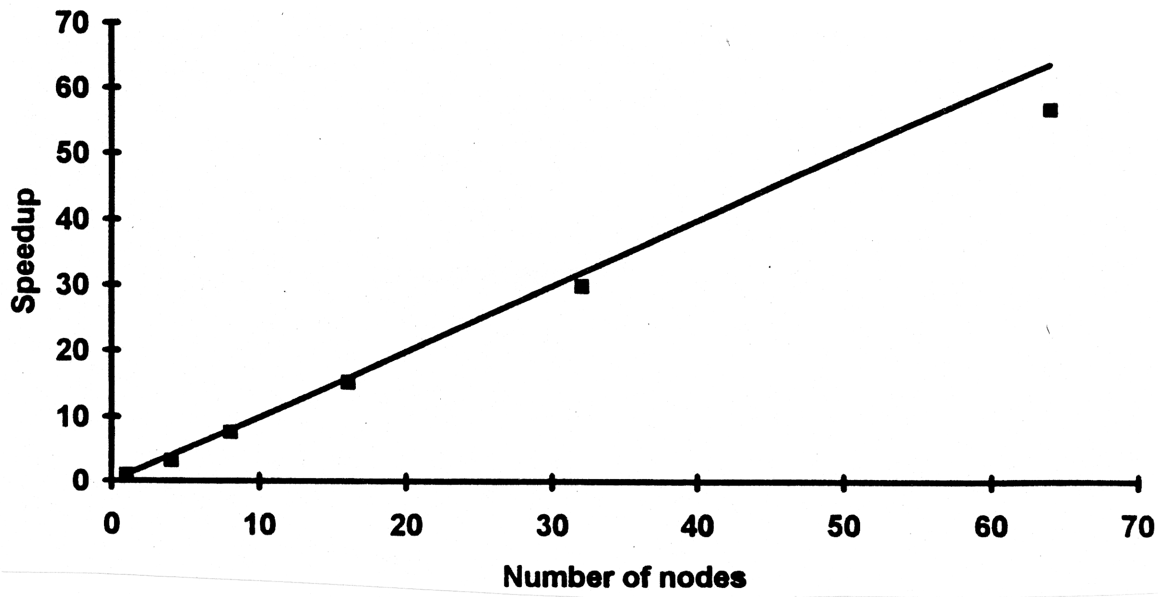


Figure 5: Speedups for various numbers of nodes.

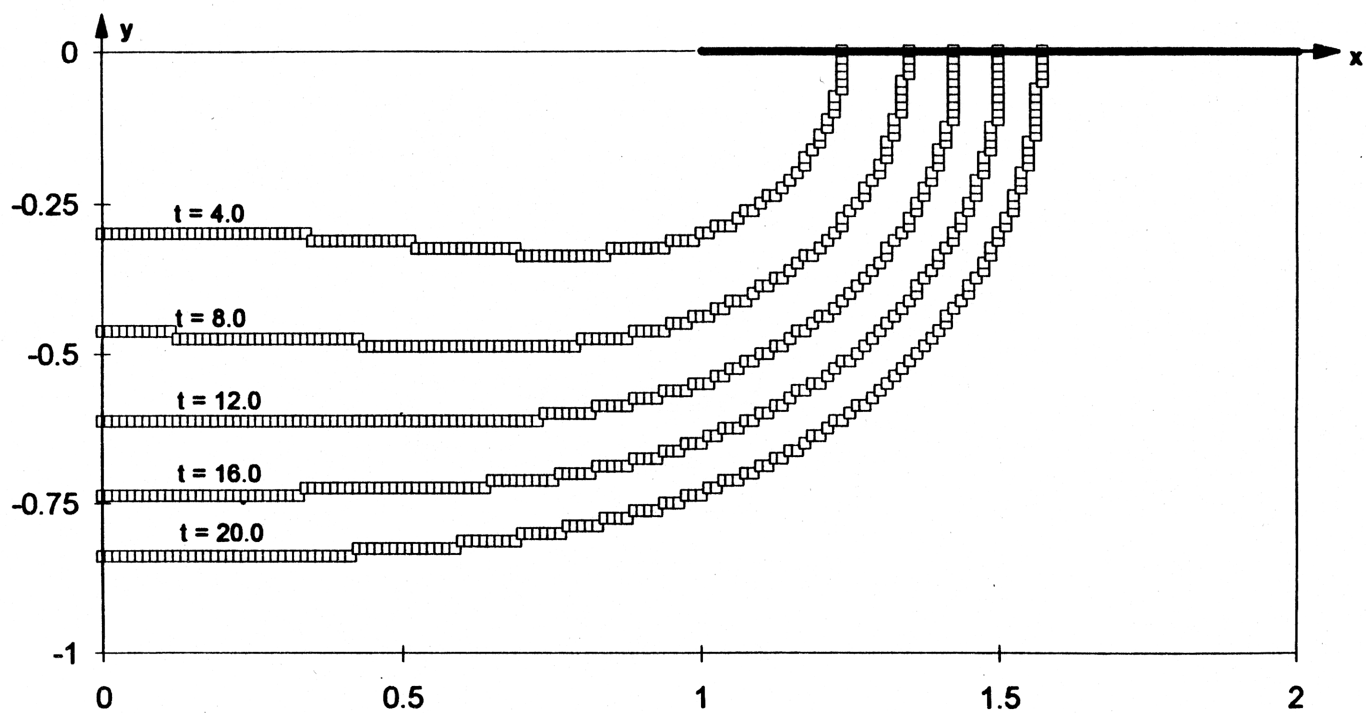


Figure 4: Moving boundary at various times.