Ordinary Differential Equations (ODEs)

ChE 132A handout

(Adapted from notes provided by Professor Sho Takatori, Department of Chemical Engineering, Stanford University, 2026.) These notes summarize material on classifying ODEs, solving common first-order and second-order problems, and introducing power-series methods for variable-coefficient equations.

1 Classifying differential equations

Before solving an equation, it helps to know what kind of equation it is. Four common classifications are:

1. ODE versus PDE
2. order
3. linear versus nonlinear
4. homogeneous versus nonhomogeneous

ODE versus PDE

ODE

One independent variable

Unknown: $f(x)$ or $f(t)$

Only ordinary derivatives appear

Example: $\dfrac{d^2 f}{dx^2} + \sin x = 0$

PDE

More than one independent variable

Unknown: $f(x, t)$ or $f(x, y, z)$

Partial derivatives appear

Example: $\dfrac{\partial f}{\partial t} = \nabla^2 f$

with $\nabla^2 = \dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2} + \dfrac{\partial^2}{\partial z^2}$

Figure 1: A first classification: ordinary differential equations involve one independent variable, while partial differential equations involve several independent variables

Definition 1.1 (Order)**.** The *order* of a differential equation is the highest derivative that appears in the equation.

Examples:

$$\frac{d^2 f}{dx^2} + x^5 = 0 \qquad \text{is second order}$$

$$\frac{df}{dt} + f^2 = \sin t \qquad \text{is first order}$$

Definition 1.2 (Linear and nonlinear equations)**.** A differential equation is *linear* if the unknown function and its derivatives appear only to the first power and are not multiplied together inside nonlinear functions. Otherwise, the equation is *nonlinear*.

Examples:

$$\frac{\mathrm{d}y}{\mathrm{d}x} + \sin y = 0 \qquad \text{nonlinear}$$
$$\frac{\mathrm{d}y}{\mathrm{d}x} + \sin(x) = 0 \qquad \text{linear}$$
$$x^3 \frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + xy = 0 \qquad \text{linear}$$
$$y\frac{\mathrm{d}y}{\mathrm{d}x} + x = 0 \qquad \text{nonlinear}$$

Definition 1.3 (Homogeneous and nonhomogeneous)**.** For a linear equation, if setting the unknown function equal to zero makes the equation identically true, then the equation is *homogeneous*. Otherwise it is *nonhomogeneous*.

Examples:

$$f'' + xf' + f^2 = 0 \qquad \text{not linear so the homogeneous label does not apply}$$
$$3x\frac{\mathrm{d}^2 f}{\mathrm{d}x^2} + \frac{\mathrm{d}f}{\mathrm{d}x} - 2f^2 = 0 \qquad \text{ODE, second order, nonlinear}$$
$$\left(\frac{\partial^4 f}{\partial x^4}\right) + \left(\frac{\partial^4 f}{\partial y^4}\right) - e^{xy} = 0 \qquad \text{PDE, fourth order, linear, nonhomogeneous}$$

2 First-order ODEs

The first-order equations in these lectures fall into three common categories:

1. linear equations solved with an integrating factor
2. separable nonlinear equations
3. exact equations

2.1 Linear first-order equations and integrating factors

Consider a linear first-order ODE in standard form

$$f'(x) + p(x)f(x) = g(x)$$

We would like the left-hand side to become the derivative of a product. Multiply by a function $F(x)$ and require

$$F(x)f'(x) + F(x)p(x)f(x) = F(x)g(x)$$
$$\frac{\mathrm{d}}{\mathrm{d}x}\big(F(x)f(x)\big) = F(x)g(x)$$

For this to work we need $F'(x) = p(x)F(x)$, so the integrating factor is

$$\frac{F'(x)}{F(x)} = p(x)$$

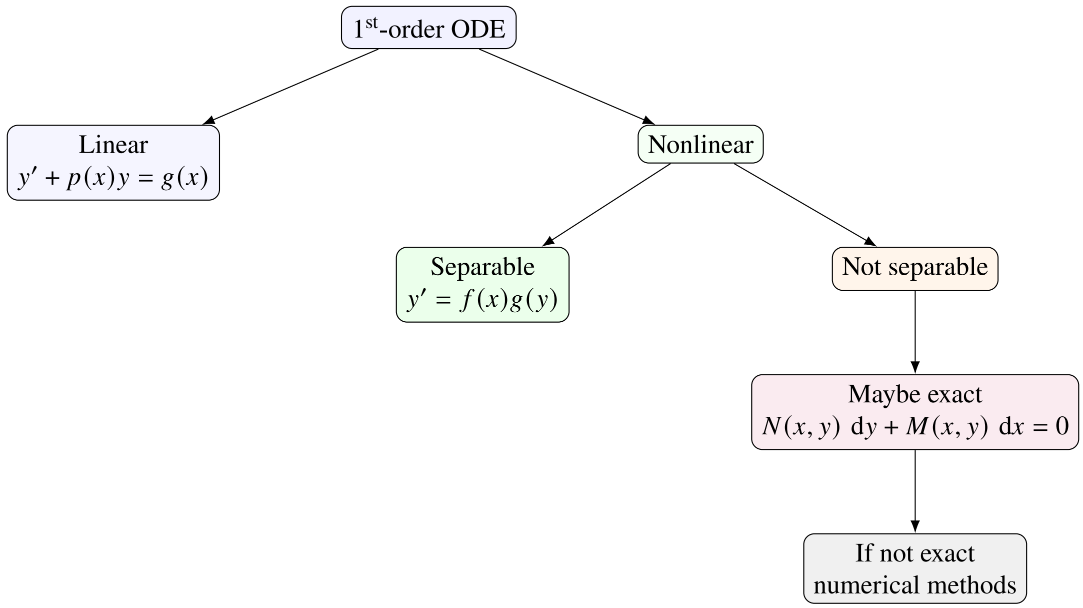

Figure 2: A practical roadmap for first-order equations

$$\ln F(x) = \int p(x)\ \mathrm{d}x$$
$$F(x) = \exp\left(\int p(x)\ \mathrm{d}x\right)$$

Integrating both sides gives the solution formula

$$F(x)f(x) = \int g(x)F(x)\ \mathrm{d}x + C$$
$$f(x) = \frac{1}{F(x)}\left[\int g(x)F(x)\ \mathrm{d}x + C\right]$$

Example 2.1 (A solvable integrating-factor problem)**.** Solve

$$\frac{\mathrm{d}u}{\mathrm{d}x} + 2u = 2 \qquad u(0) = 0$$

Here $p(x) = 2$, so the integrating factor is

$$F(x) = e^{\int 2\ \mathrm{d}x} = e^{2x}$$

Multiplying through by $F(x)$ gives

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(e^{2x}u(x)\right) = 2e^{2x}$$

Integrating,

$$e^{2x}u(x) = e^{2x} + C$$
$$u(x) = 1 + Ce^{-2x}$$

Applying the initial condition $u(0) = 0$ gives $C = -1$, so

$$u(x) = 1 - e^{-2x}$$

Example 2.2 (A problem whose final integral is not elementary)**.** Solve

$$x^2 \frac{\mathrm{d}u}{\mathrm{d}x} + 2u = 5x$$

First rewrite the equation in standard form

$$\frac{\mathrm{d}u}{\mathrm{d}x} + \frac{2}{x^2} u = \frac{5}{x}$$

The integrating factor is

$$F(x) = \exp\left(\int \frac{2}{x^2} \, \mathrm{d}x \right) = e^{-2/x}$$

Therefore

$$\frac{\mathrm{d}}{\mathrm{d}x} \left(e^{-2/x} u(x) \right) = \frac{5}{x} e^{-2/x}$$

Integrating gives

$$u(x) = e^{2/x} \left[\int \frac{5}{x} e^{-2/x} \, \mathrm{d}x + C \right]$$

The last integral is typically evaluated numerically.

2.2 Separable equations

A nonlinear first-order equation is *separable* if it can be written as

$$\frac{\mathrm{d}y}{\mathrm{d}x} = f(x) g(y)$$

Then variables can be separated

$$\frac{\mathrm{d}y}{g(y)} = f(x) \, \mathrm{d}x$$

$$\int \frac{\mathrm{d}y}{g(y)} = \int f(x) \, \mathrm{d}x$$

Example 2.3 (A separable equation)**.** Solve

$$3y^2 \frac{\mathrm{d}y}{\mathrm{d}x} = \cos x$$

Separate variables and integrate

$$3y^2 \, \mathrm{d}y = \cos x \, \mathrm{d}x$$

$$\int 3y^2 \, \mathrm{d}y = \int \cos x \, \mathrm{d}x$$

$$y^3 = \sin x + C$$

Hence

$$y(x) = \left(\sin x + C \right)^{1/3}$$

2.3 Exact equations

Consider an equation written as

$$N(x, y)\ \mathrm{d}y + M(x, y)\ \mathrm{d}x = 0$$

We say the equation is exact if there exists a potential function $\phi(x, y)$ such that

$$\frac{\partial \phi}{\partial y} = N(x, y) \qquad \frac{\partial \phi}{\partial x} = M(x, y)$$

Then

$$\mathrm{d}\phi = \frac{\partial \phi}{\partial x}\ \mathrm{d}x + \frac{\partial \phi}{\partial y}\ \mathrm{d}y = M(x, y)\ \mathrm{d}x + N(x, y)\ \mathrm{d}y$$

So the ODE becomes

$$\mathrm{d}\phi = 0$$

which implies

$$\phi(x, y) = C$$

If M and N are continuously differentiable, the exactness test is

$$\frac{\partial N}{\partial x} = \frac{\partial M}{\partial y}$$

A practical procedure is:

1. write the equation in the form $N(x, y)\ \mathrm{d}y + M(x, y)\ \mathrm{d}x = 0$
2. check whether $N_x = M_y$
3. integrate N with respect to y or M with respect to x
4. determine the missing function of the other variable
5. apply the initial or boundary condition and solve for y

Example 2.4 (An exact equation)**.** Solve

$$(2 + x^2 y)\frac{\mathrm{d}y}{\mathrm{d}x} + xy^2 = 0 \qquad y(1) = 2$$

Write the equation as

$$(2 + x^2 y)\ \mathrm{d}y + xy^2\ \mathrm{d}x = 0$$

Thus

$$N(x, y) = 2 + x^2 y \qquad M(x, y) = xy^2$$

Check exactness

$$\frac{\partial N}{\partial x} = 2xy \qquad \frac{\partial M}{\partial y} = 2xy$$

So the equation is exact. Now integrate N with respect to y

$$\phi(x, y) = \int (2 + x^2 y)\ \mathrm{d}y + f(x) = 2y + \frac{x^2 y^2}{2} + f(x)$$

Differentiate with respect to x and match M

$$\frac{\partial \phi}{\partial x} = xy^2 + f'(x) = M(x, y) = xy^2$$

Hence $f'(x) = 0$, so $f(x)$ is constant and may be absorbed into C. Therefore

$$\phi(x, y) = 2y + \frac{(xy)^2}{2} = C$$

Use $y(1) = 2$

$$2(2) + \frac{(1 \cdot 2)^2}{2} = 6$$

Thus the implicit solution is

$$2y + \frac{(xy)^2}{2} = 6$$

Solving for y gives the branch satisfying $y(1) = 2$

$$y(x) = \frac{-2 + 2\sqrt{1 + 3x^2}}{x^2}$$

3 Second-order linear ODEs

The next major class in the notes is the linear second-order equation

$$u'' + p(x)u' + q(x)u = g(x)$$

When p and q are constants, many problems can be solved analytically. When coefficients vary with x, power-series and Laplace-transform methods become useful.

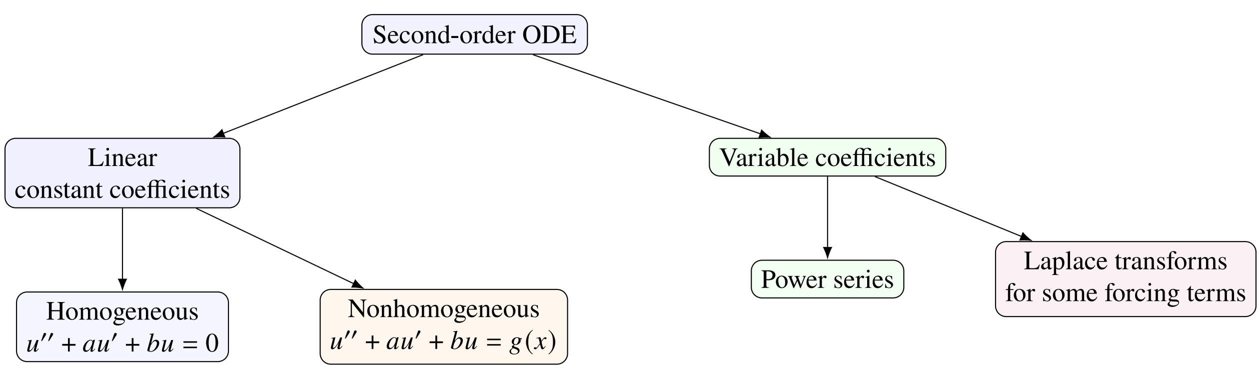

Figure 3: Roadmap for the second-order material in these lectures

3.1 Homogeneous constant-coefficient equations

Consider

$$u'' + au' + bu = 0$$

We seek solutions of the form $u(x) = Ae^{\lambda x}$. Substituting gives

$$\lambda^2 Ae^{\lambda x} + a\lambda Ae^{\lambda x} + bAe^{\lambda x} = 0$$

Since $Ae^{\lambda x} \neq 0$, we obtain the *characteristic equation*

$$\lambda^2 + a\lambda + b = 0$$

The roots determine the solution form.

Case 1: distinct real roots

If $\lambda_1 \neq \lambda_2$ are real, then

$$u_1(x) = e^{\lambda_1 x} \qquad u_2(x) = e^{\lambda_2 x}$$
$$u_h(x) = c_1 e^{\lambda_1 x} + c_2 e^{\lambda_2 x}$$

Case 2: repeated real root

If $\lambda_1 = \lambda_2 = \lambda$, then

$$u_1(x) = e^{\lambda x} \qquad u_2(x) = xe^{\lambda x}$$
$$u_h(x) = c_1 e^{\lambda x} + c_2 xe^{\lambda x}$$

Case 3: complex roots

If the roots are $\lambda_{1,2} = \alpha \pm i\beta$, then

$$e^{(\alpha+i\beta)x} = e^{\alpha x} e^{i\beta x}$$

Using Euler's formula (see Figure 4),

$$e^{i\theta} = \cos\theta + i\sin\theta$$

we obtain the real solution basis

$$u_1(x) = e^{\alpha x}\cos(\beta x) \qquad u_2(x) = e^{\alpha x}\sin(\beta x)$$
$$u_h(x) = e^{\alpha x}\big(c_1\cos(\beta x) + c_2\sin(\beta x)\big)$$

Example 3.1 (Distinct real roots)**.** Solve

$$u'' + 5u' + 6u = 0$$

The characteristic equation is

$$\lambda^2 + 5\lambda + 6 = 0$$

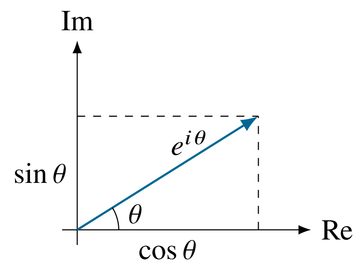

Figure 4: The point $e^{i\theta} = \cos\theta + i\sin\theta$ on the complex plane

$$(\lambda+2)(\lambda+3) = 0$$

So $\lambda_1 = -2$ and $\lambda_2 = -3$, and the solution is

$$u_h(x) = c_1 e^{-2x} + c_2 e^{-3x}$$

Example 3.2 (Repeated root)**.** Solve

$$u'' + 6u' + 9u = 0$$

The characteristic equation is

$$\lambda^2 + 6\lambda + 9 = 0$$
$$(\lambda+3)^2 = 0$$

Hence $\lambda = -3$ is repeated and

$$u_h(x) = c_1 e^{-3x} + c_2 x e^{-3x}$$

Example 3.3 (Complex roots)**.** Solve

$$u'' + 2u' + 5u = 0$$

The characteristic equation is

$$\lambda^2 + 2\lambda + 5 = 0$$
$$\lambda = -1 \pm 2i$$

Therefore

$$u_h(x) = e^{-x}\big(c_1\cos(2x) + c_2\sin(2x)\big)$$

3.2 Linear independence and the Wronskian

Two candidate solutions u_1 and u_2 are linearly independent if the Wronskian

$$W(x) = \begin{vmatrix} u_1 & u_2 \\ u_1' & u_2' \end{vmatrix} = u_1 u_2' - u_1' u_2$$

is nonzero.

For distinct exponentials,

$$u_1 = e^{\lambda_1 x} \qquad u_2 = e^{\lambda_2 x}$$

$$W(x) = \begin{vmatrix} e^{\lambda_1 x} & e^{\lambda_2 x} \\ \lambda_1 e^{\lambda_1 x} & \lambda_2 e^{\lambda_2 x} \end{vmatrix} = (\lambda_2 - \lambda_1) e^{(\lambda_1 + \lambda_2) x}$$

So $W(x) \neq 0$ for all x exactly when $\lambda_1 \neq \lambda_2$.

For a repeated root with basis $u_1 = e^{\lambda x}$ and $u_2 = x e^{\lambda x}$,

$$W(x) = \begin{vmatrix} e^{\lambda x} & x e^{\lambda x} \\ \lambda e^{\lambda x} & (1 + \lambda x) e^{\lambda x} \end{vmatrix} = e^{2\lambda x}$$

which is never zero.

3.3 Nonhomogeneous equations

For

$$u'' + au' + bu = f(x)$$

the general solution is

$$u(x) = u_h(x) + u_p(x)$$

where u_h solves the homogeneous equation and u_p is any particular solution.

Method of undetermined coefficients

This method works when the forcing term has a standard form. A useful rule of thumb is to choose u_p to have the same general shape as the right-hand side, then multiply by extra factors of x if needed to avoid overlap with u_h.

Right-hand side $f(x)$	Trial particular solution $u_p(x)$
Polynomial of degree n	$a_0 + a_1 x + \cdots + a_n x^n$
Ce^{kx}	Ae^{kx}
$C\cos(kx)$ or $C\sin(kx)$	$A\cos(kx) + B\sin(kx)$
Resonance with one homogeneous root	Multiply the trial by x
Resonance with a repeated root	Multiply the trial by x^2

Example 3.4 (Undetermined coefficients)**.** Solve

$$u'' + u = x^2$$

The homogeneous equation is

$$u_h'' + u_h = 0$$

The characteristic equation is

$$\lambda^2 + 1 = 0$$

so

$$u_h(x) = c_1 \cos x + c_2 \sin x$$

Since the right-hand side is a quadratic polynomial, try

$$u_p(x) = a_0 + a_1 x + a_2 x^2$$

Then

$$u_p'(x) = a_1 + 2a_2 x \qquad u_p''(x) = 2a_2$$

Substituting into the ODE,

$$2a_2 + a_0 + a_1 x + a_2 x^2 = x^2$$

Matching coefficients gives

$$a_2 = 1 \qquad a_1 = 0 \qquad 2a_2 + a_0 = 0$$

so $a_0 = -2$. Thus

$$u_p(x) = x^2 - 2$$

and the total solution is

$$u(x) = c_1 \cos x + c_2 \sin x + x^2 - 2$$

Variation of parameters

When undetermined coefficients does not apply, variation of parameters is a general method.

For the normalized equation

$$u'' + p(x)u' + q(x)u = f(x)$$

assume a particular solution of the form

$$u_p(x) = V_1(x)u_1(x) + V_2(x)u_2(x)$$

where u_1 and u_2 are linearly independent homogeneous solutions. Then

$$V_1'(x) = -\frac{u_2(x) f(x)}{W(x)} \qquad V_2'(x) = \frac{u_1(x) f(x)}{W(x)}$$

with

$$W(x) = u_1 u_2' - u_1' u_2$$

Integrating gives

$$V_1(x) = -\int \frac{u_2(x) f(x)}{W(x)}\,\mathrm{d}x \qquad V_2(x) = \int \frac{u_1(x) f(x)}{W(x)}\,\mathrm{d}x$$

Example 3.5 (Variation of parameters for the same problem)**.** Solve again

$$u'' + u = x^2$$

Here we already know that

$$u_1(x) = \cos x \qquad u_2(x) = \sin x \qquad W(x) = 1$$

Therefore

$$V_1(x) = -\int x^2 \sin x \, \mathrm{d}x \qquad V_2(x) = \int x^2 \cos x \, \mathrm{d}x$$

Evaluating these integrals by parts gives

$$V_1(x) = 2x \sin x + (2 - x^2) \cos x$$
$$V_2(x) = 2x \cos x + (x^2 - 2) \sin x$$

Hence

$$u_p(x) = V_1 u_1 + V_2 u_2$$
$$u_p(x) = \left[2x \sin x + (2 - x^2) \cos x\right] \cos x + \left[2x \cos x + (x^2 - 2) \sin x\right] \sin x$$
$$u_p(x) = x^2 - 2$$

which agrees with the result from undetermined coefficients.

4 Power-series methods and radius of convergence

When the coefficients are not constant, one common idea is to represent the solution as a power series and then solve for its coefficients.

4.1 Power-series representations

A regular power series about $x = a$ has the form

$$f(x) = \sum_{n=0}^{\infty} b_n (x - a)^n$$

A familiar example is the Taylor series for $\cos x$ about $x = 0$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots$$
$$\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$$

(see Figure 5)

For a variable-coefficient ODE we can try

$$y(x) = \sum_{n=0}^{\infty} c_n (x - a)^n$$

The basic steps are:

1. assume a series form for $y(x)$
2. differentiate term by term to get $y'(x)$ and $y''(x)$
3. substitute into the differential equation
4. match powers of $(x - a)$ to determine the coefficients c_n

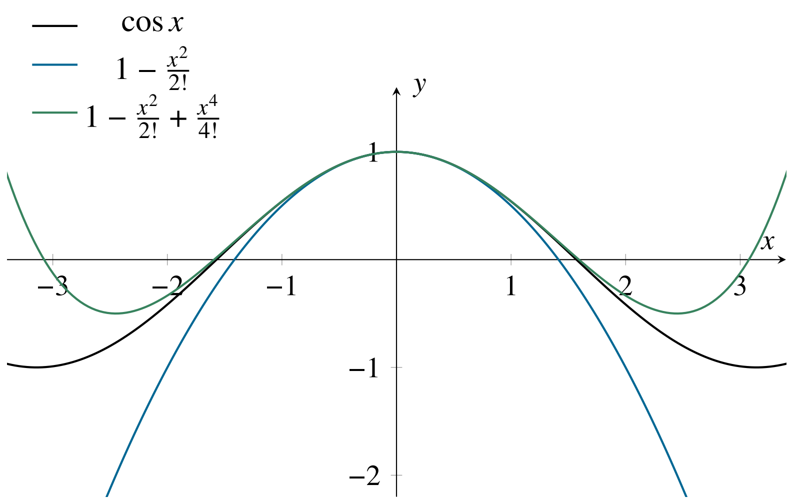

Figure 5: A power series gives polynomial approximations that are most accurate near the expansion point

4.2 Radius of convergence

A power series does not necessarily converge for every x. The *radius of convergence* tells us where the series is valid.

For

$$f(x) = \sum_{n=0}^{\infty} b_n (x-a)^n$$

if the ratio limit exists, define

$$\frac{1}{R} = \lim_{n\to\infty} \left| \frac{b_{n+1}}{b_n} \right|$$

Then:

- if $R = 0$, the series diverges for every $x \neq a$
- if $0 < R < \infty$, the series converges for $|x-a| < R$ and diverges for $|x-a| > R$
- if $R = \infty$, the series converges for all x

A useful geometric interpretation is that R is the distance from the expansion point to the nearest singularity of the function in the complex plane.

Example 4.1 (Simple poles)**.** For

$$f(x) = \frac{1}{1-x}$$

there is a singularity at $x = 1$, so the power series about $x = 0$ has radius of convergence

$$R = 1$$

Example 4.2 (Nearest singularity in the complex plane)**.** For

$$f(x) = \frac{1}{(x-6)(x^2+9)}$$

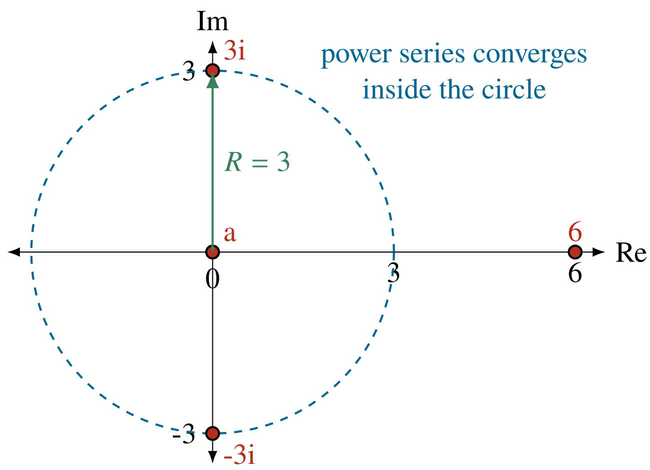

Figure 6: The radius of convergence is the distance to the nearest singularity in the complex plane

The singularities are at

$$x = 6 \qquad x = 3i \qquad x = -3i$$

About $x = 0$, the nearest singularities are $\pm 3i$, which are distance 3 away, so

$$R = 3$$

(see Figure 6)

Example 4.3 (Entire functions)**.** The functions

$$e^x \qquad \sin x \qquad \cos x$$

are analytic everywhere in the complex plane and have no singularities, so their power series have

$$R = \infty$$

4.3 Ordinary and singular points

For the equation

$$y'' + p(x)y' + q(x)y = g(x)$$

there are two important types of expansion points.

Definition 4.1 (Ordinary point)**.** A point $x = a$ is an *ordinary point* if the coefficient functions $p(x)$, $q(x)$, and $g(x)$ are analytic at $x = a$.

Definition 4.2 (Singular point)**.** A point $x = a$ is a *singular point* if at least one of the coefficient functions fails to be analytic at $x = a$.

At an ordinary point, power-series methods behave especially well:

Summary 4.3 (Power-series theorem at an ordinary point)**.** If $x = a$ is an ordinary point of $p(x)$, $q(x)$, and $g(x)$, then there is a power-series solution about $x = a$ of the form

$$y(x) = \sum_{n=0}^{\infty} b_n (x - a)^n$$

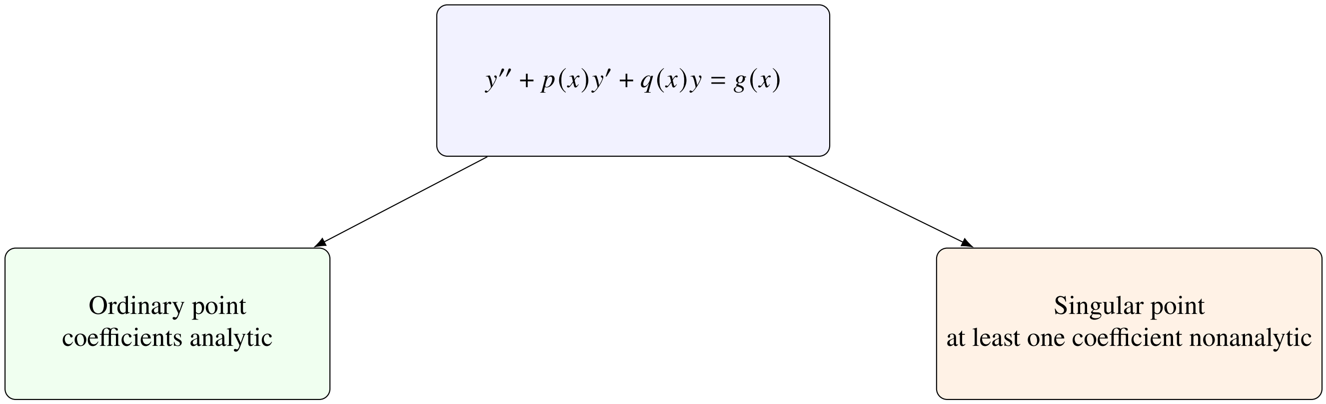

Figure 7: Ordinary versus singular points for a second-order linear ODE

and the series converges for at least all x satisfying

$$|x - a| < R$$

where R is the distance from a to the nearest singularity of the coefficient functions.

Closing summary

These lectures cover a useful progression of ideas:

- classify the equation before trying to solve it
- use integrating factors for first-order linear equations
- separate variables when possible
- check for exactness in nonlinear first-order equations
- solve constant-coefficient second-order equations from the characteristic polynomial
- use undetermined coefficients or variation of parameters for forcing terms
- use power-series methods when coefficients vary with x
- estimate where the series is valid by finding the radius of convergence

Exercises

Exercise 1 (Classifying differential equations).

For each equation below, state: (i) ODE or PDE, (ii) order, and (iii) linear or nonlinear. For linear equations also state whether the equation is homogeneous or nonhomogeneous.

(a) $\dfrac{d^2u}{dx^2} + 4\dfrac{du}{dx} + 4u = 0$

(b) $\dfrac{df}{dt} + f^2 = \sin t$

(c) $\dfrac{\partial c}{\partial t} = D\,\dfrac{\partial^2 c}{\partial x^2}$ (D is a positive constant)

(d) $y'' + x\,y' + 2y = 0$

(e) $y'' + y = 3\cos x$

(f) $y\,\dfrac{dy}{dx} + x = 0$

(g) For equations (a), (d), (e), and (f), identify which solution method from the handout applies: integrating factor, separation of variables, characteristic equation (constant-coefficient homogeneous), undetermined coefficients, or power series.

Exercise 2 (First-order ODEs).

(a) Solve

$$\frac{dy}{dx} + \frac{2}{x}\,y = 4x \qquad y(1) = 3$$

(b) Solve

$$\frac{dy}{dx} = y\sin x \qquad y(0) = 3$$

(c) Show that

$$(3x^2y + 2)\,dx + (x^3 + 4y)\,dy = 0$$

is exact, find a potential function $\phi(x, y)$, and find the implicit solution satisfying $y(0) = 1$.

Exercise 3 (Second-order constant-coefficient equations).

(a) Solve $u'' - u' - 6u = 0$ with $u(0) = 1$ and $u'(0) = 0$.

(b) Write the general solution of $u'' + 6u' + 9u = 0$.

(c) Solve $u'' - 4u' + 13u = 0$ with $u(0) = 0$ and $u'(0) = 3$.

Exercise 4 (Second-order constant-coefficient equations: nonhomogeneous).

Find the general solution of

$$u'' - 3u' + 2u = 4e^x$$

Note that e^x also appears in the homogeneous solution; how does this affect the trial form for the particular solution?

Exercise 5 (Power-series solution and radius of convergence).

(a) Write $(x^2 - 9)\, y'' + 3x\, y' + y = 0$ in standard form $y'' + p(x)y' + q(x)y = 0$. Identify all singular points and, using the geometric interpretation of the radius of convergence as the distance to the nearest singularity, state the radius of convergence of a power-series solution expanded about $x = 0$.

(b) For the equation

$$y'' + x\, y' + y = 0$$

is $x = 0$ an ordinary or a singular point? What radius of convergence do you expect for a power-series solution about $x = 0$?

(c) Assume $y(x) = \sum_{n=0}^{\infty} c_n x^n$ and substitute into $y'' + xy' + y = 0$. Shift indices as needed to collect all terms under a single power of x and derive a recurrence relation expressing c_{n+2} in terms of c_n.

(d) Use the recurrence relation to write the first four nonzero terms of each of the two linearly independent solutions: the even series ($c_0 = 1$, $c_1 = 0$) and the odd series ($c_0 = 0$, $c_1 = 1$). Does the radius of convergence agree with your answer in part (b)? Bonus: verify that the even series is the Taylor expansion of $e^{-x^2/2}$.

Exercise 6 (Batch reactor kinetics).

A first-order irreversible reaction $A \to B$ occurs in a batch reactor. The molar concentration $c_A(t)$ satisfies

$$\frac{dc_A}{dt} = -kc_A \qquad c_A(0) = c_{A0}$$

where k is the rate constant and c_{A0} is the initial concentration.

(a) Classify the equation: state whether it is an ODE or PDE, its order, and whether it is linear or nonlinear. For linear equations state whether it is homogeneous or nonhomogeneous. Identify the two first-order solution methods from the handout that both apply here.

(b) Solve the equation by separating variables.

(c) Show that the integrating-factor method gives the same result.