Laplace Transform Table

Larger table of Laplace transforms.

	$f(t)$	$\overline{f}(s)$	**page**
1	$\alpha f(t) + \beta g(t)$	$\alpha \overline{f}(s) + \beta \overline{g}(s)$	106
2	$\dfrac{\mathrm{d}f(t)}{\mathrm{d}t}$	$s\overline{f}(s) - f(0)$	107
3	$\dfrac{\mathrm{d}^2 f(t)}{\mathrm{d}t^2}$	$s^2\overline{f}(s) - sf(0) - f'(0)$	107
4	$\dfrac{\mathrm{d}^n f(t)}{\mathrm{d}t^n}$	$s^n\overline{f}(s) - \sum_{i=1}^{n} s^{n-i} f^{(i-1)}(0)$	107
5	$\int_0^t f(t')\ \mathrm{d}t'$	$\dfrac{1}{s}\overline{f}(s)$	107
6	$t^n f(t)$	$(-1)^n \dfrac{\mathrm{d}^n \overline{f}(s)}{\mathrm{d}s^n}$	107, 228
7	$f(t-a)H(t-a)$	$e^{-as}\overline{f}(s)$	108
8	$e^{at} f(t)$	$\overline{f}(s-a)$	108
9	$\int_0^t f(t')g(t-t')\ \mathrm{d}t'$	$\overline{f}(s)\overline{g}(s)$	108, 227
10	$\lim_{t\to 0^+} f(t)$ (initial value theorem)	$\lim_{s\to\infty} s\overline{f}(s)$	108, 227
11	$\lim_{t\to\infty} f(t)$ (final value theorem)	$\lim_{s\to 0} s\overline{f}(s)^\dagger$	108, 227
12	$H(t)$	$\dfrac{1}{s}$	109
13	$\delta(t)$	1	115
14	$\delta^{(n)}(t), \quad n \geq 0$	s^n	115
15	t	$\dfrac{1}{s^2}$	109
16	$t^n, \quad n > -1$	$\dfrac{\Gamma(n+1)}{s^{n+1}}$	109
17	e^{at}	$\dfrac{1}{s-a}$	109
18	$e^{At}, \quad A \in \mathbb{R}^{n\times n}$	$(sI - A)^{-1}$	112
19	te^{at}	$\dfrac{1}{(s-a)^2}$	109

continued on next page

continued from previous page

	$f(t)$	$\overline{f}(s)$	**page**
20	$\sin\omega t$	$\dfrac{\omega}{s^2+\omega^2}$	109
21	$\cos\omega t$	$\dfrac{s}{s^2+\omega^2}$	109
22	$\sinh\omega t$	$\dfrac{\omega}{s^2-\omega^2}$	109
23	$\cosh\omega t$	$\dfrac{s}{s^2-\omega^2}$	109
24	$e^{at}\sin\omega t$	$\dfrac{\omega}{(s-a)^2+\omega^2}$	109
25	$e^{at}\cos\omega t$	$\dfrac{s-a}{(s-a)^2+\omega^2}$	109
26	$\sum_{n=1}^{m}\dfrac{\overline{p}(s_n)}{\overline{q}'(s_n)}e^{s_n t}, \quad \overline{q}(s_n)$ simple zero	$\dfrac{\overline{p}(s)}{\overline{q}(s)}$	315
27	$\sum_{n=1}^{m}e^{s_n t}\sum_{i=1}^{r_n}a_{ni}t^{i-1}, \quad \overline{q}(s_n)$ zero of order r_n	$\dfrac{\overline{p}(s)}{\overline{q}(s)}^*$	316
28	$\dfrac{k}{2\sqrt{\pi t^3}}\,e^{-k^2/(4t)}$	$e^{-k\sqrt{s}}, \quad k>0$	337
29	$\dfrac{1}{\sqrt{\pi t}}\,e^{-k^2/(4t)}$	$\dfrac{e^{-k\sqrt{s}}}{\sqrt{s}}, \quad k>0$	337
30	$\operatorname{erfc}\left(\dfrac{k}{2\sqrt{t}}\right)^{\ddagger}$	$\dfrac{e^{-k\sqrt{s}}}{s}, \quad k>0$	337
31	$\dfrac{e^{\alpha t}}{2\sqrt{\alpha}}\left\{e^{-k\sqrt{\alpha}}\operatorname{erfc}\left(\dfrac{k}{2\sqrt{t}}-\sqrt{\alpha t}\right)-e^{k\sqrt{\alpha}}\operatorname{erfc}\left(\dfrac{k}{2\sqrt{t}}+\sqrt{\alpha t}\right)\right\}$	$\dfrac{e^{-k\sqrt{s}}}{(s-\alpha)\sqrt{s}}, \quad k>0$	337
32	$\dfrac{2}{\sqrt{\pi}}\sqrt{t}\,e^{-k^2/(4t)}-k\ \operatorname{erfc}\left(\dfrac{k}{2\sqrt{t}}\right)$	$\dfrac{e^{-k\sqrt{s}}}{s\sqrt{s}}, \quad k>0$	551
33	$\dfrac{e^{\alpha t}}{2}\left\{e^{-k\sqrt{\alpha}}\operatorname{erfc}\left(\dfrac{k}{2\sqrt{t}}-\sqrt{\alpha t}\right)+e^{k\sqrt{\alpha}}\operatorname{erfc}\left(\dfrac{k}{2\sqrt{t}}+\sqrt{\alpha t}\right)\right\}$	$\dfrac{e^{-k\sqrt{s}}}{s-\alpha}, \quad k>0$	351
34	$\dfrac{1}{2t}e^{-k^2/(4t)}$	$K_0(k\sqrt{s}), \quad k>0$	338
35	$\dfrac{1}{k}e^{-k^2/(4t)}$	$\dfrac{K_1(k\sqrt{s})}{\sqrt{s}}, \quad k>0$	338
36	$\dfrac{\sinh(x\sqrt{k})}{\sinh\sqrt{k}}-2\sum_{n=1}^{\infty}\dfrac{(-1)^{n+1}n\pi}{n^2\pi^2+k}\sin(n\pi x)e^{-(n^2\pi^2+k)t}$	$\dfrac{\sinh(x\sqrt{s+k})}{s\ \sinh\sqrt{s+k}}$	321
37	$1-2\sum_{n=1}^{\infty}\dfrac{(-1)^{n+1}}{n\pi x}\sin(n\pi x)e^{-n^2\pi^2 t}$	$\dfrac{\sinh(x\sqrt{s})}{xs\ \sinh\sqrt{s}}$	342
38	$1+2\sum_{n=1}^{\infty}(-1)^n\cos(n\pi x)e^{-n^2\pi^2 t}$	$\dfrac{\cosh(x\sqrt{s})}{\sqrt{s}\ \sinh\sqrt{s}}$	353
39	$\dfrac{\cosh(x\sqrt{k})}{\cosh\sqrt{k}}-2\sum_{n=0}^{\infty}\dfrac{(-1)^n(n+1/2)\pi}{(n+1/2)^2\pi^2+k}\cos\big((n+1/2)\pi x\big)e^{-((n+1/2)^2\pi^2+k)t}$	$\dfrac{\cosh(x\sqrt{s+k})}{s\ \cosh\sqrt{s+k}}$	340
40	$1-2\sum_{n=1}^{\infty}\dfrac{1}{\alpha_n J_1(\alpha_n)}J_0(\alpha_n x)e^{-\alpha_n^2 t}, \quad J_0(\alpha_n)=0$	$\dfrac{I_0(x\sqrt{s})}{sI_0(\sqrt{s})}$	342

continued on next page

continued from previous page

	$f(t)$	$\overline{f}(s)$	**page**
41	$2\sum_{n=1}^{\infty}(-1)^{n+1}\sin(n\pi a)\sin(n\pi b)\cos(n\pi t)$	$\dfrac{\sinh(as)\sinh(bs)}{\sinh s}$	322
42	$2\sum_{n=1}^{\infty}\dfrac{(-1)^{n+1}}{n\pi}\sin(n\pi a)\sin(n\pi b)\sin(n\pi t)$	$\dfrac{\sinh(as)\sinh(bs)}{s\ \sinh s}$	348

† Final value exists if and only if $s\overline{f}(s)$ is bounded for $\mathrm{Re}(s) \geq 0$.

* $a_{ni} = \frac{\Phi^{(r_n-i)}(s_n)}{(r_n-i)!(i-1)!}$ $\quad \Phi(s) = (s - s_n)^{r_n}\overline{p}(s)/\overline{q}(s)$.

‡ $\mathrm{erfc}(z) = \dfrac{2}{\sqrt{\pi}}\displaystyle\int_z^{\infty} e^{-u^2}\,du$.

Del Operator in Common Coordinate Systems

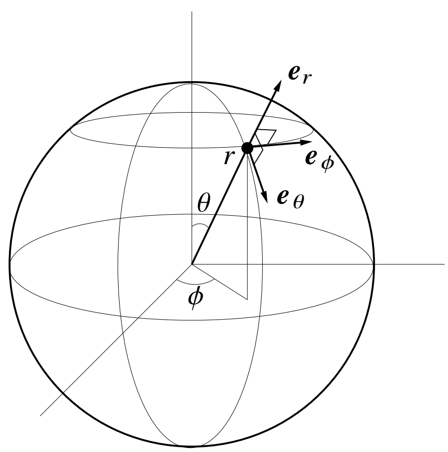

Figure 1: Orthonormal unit vectors $\boldsymbol{e}_r$, $\boldsymbol{e}_\theta$, $\boldsymbol{e}_\phi$ in spherical coordinates (r, θ, ϕ).

Cartesian	$\nabla = \boldsymbol{e}_x\dfrac{\partial}{\partial x} + \boldsymbol{e}_y\dfrac{\partial}{\partial y} + \boldsymbol{e}_z\dfrac{\partial}{\partial z}$	$\nabla^2 = \dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2} + \dfrac{\partial^2}{\partial z^2}$
Cylindrical	$\nabla = \boldsymbol{e}_r\dfrac{\partial}{\partial r} + \boldsymbol{e}_\theta\dfrac{1}{r}\dfrac{\partial}{\partial \theta} + \boldsymbol{e}_z\dfrac{\partial}{\partial z}$	$\nabla^2 = \dfrac{1}{r}\dfrac{\partial}{\partial r}\left(r\dfrac{\partial}{\partial r}\right) + \dfrac{1}{r^2}\dfrac{\partial^2}{\partial \theta^2} + \dfrac{\partial^2}{\partial z^2}$
Spherical	$\nabla = \boldsymbol{e}_r\dfrac{\partial}{\partial r} + \boldsymbol{e}_\theta\dfrac{1}{r}\dfrac{\partial}{\partial \theta} + \boldsymbol{e}_\phi\dfrac{1}{r\sin\theta}\dfrac{\partial}{\partial \phi}$	$\nabla^2 = \dfrac{1}{r^2}\dfrac{\partial}{\partial r}\left(r^2\dfrac{\partial}{\partial r}\right) + \dfrac{1}{r^2\sin\theta}\dfrac{\partial}{\partial \theta}\left(\sin\theta\dfrac{\partial}{\partial \theta}\right) + \dfrac{1}{r^2\sin^2\theta}\dfrac{\partial^2}{\partial \phi^2}$