

## Homework 1 — Multiple Choice

ChE 132A

### Exercise 1 (Trigonometric and hyperbolic functions and their zeros).

- (a) Make a plot of  $\cosh x$  and  $\sinh x$  for real-valued  $x$ .
- (A)  $\cosh 0 = 0$  and  $\sinh 0 = 1$
  - (B)  $\cosh 0 = 1$  and  $\sinh 0 = 0$
  - (C)  $\cosh 0 = e$  and  $\sinh 0 = e$
  - (D)  $\cosh 0 = 1/2$  and  $\sinh 0 = 1/2$
- (b) Using Euler's formula, derive the relationships  $\cosh iz = \cos z$  and  $\sinh iz = i \sin z$ . Which of the following correctly states  $\sinh(iz)$ ?
- (A)  $\sinh(iz) = \cos z$
  - (B)  $\sinh(iz) = i \cos z$
  - (C)  $\sinh(iz) = \sin z$
  - (D)  $\sinh(iz) = i \sin z$
- (c) In the complex plane, sketch the locations of all zeros of  $\sin z$  and  $\cos z$ . The zeros of  $\cos z$  are located at  $z_n =$
- (A)  $n\pi, \quad n = 0, \pm 1, \pm 2, \dots$
  - (B)  $\frac{n\pi}{2}, \quad n = 0, \pm 1, \pm 2, \dots$
  - (C)  $\frac{(2n+1)\pi}{2}, \quad n = 0, \pm 1, \pm 2, \dots$
  - (D)  $(2n+1)\pi, \quad n = 0, \pm 1, \pm 2, \dots$
- (d) Using the relationship between trigonometric and hyperbolic functions, find all zeros of  $\cosh z$  and  $\sinh z$ . The zeros of  $\cosh z$  are located at  $z_n =$
- (A)  $n\pi i, \quad n = 0, \pm 1, \pm 2, \dots$
  - (B)  $\frac{n\pi}{2} i, \quad n = 0, \pm 1, \pm 2, \dots$
  - (C)  $\frac{(2n+1)\pi i}{2}, \quad n = 0, \pm 1, \pm 2, \dots$
  - (D)  $(2n+1)\pi i, \quad n = 0, \pm 1, \pm 2, \dots$
- (e) Does  $e^z$  have any zeros in the (finite) complex plane?
- (A) Yes; there is one zero at  $z = 0$
  - (B) Yes; the zeros are at  $z = i(2n+1)\pi, n = 0, \pm 1, \dots$
  - (C) Yes; the zeros are at  $z = \ln 0 = -\infty$  along the real axis
  - (D) No;  $e^z$  has no zeros anywhere in the finite complex plane

**Exercise 2 (Matrix inverse and linearly independent columns).**

The  $2 \times 2$  matrix

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

(a) For what values of the entries  $a_{ij}$  is this matrix invertible?

- (A)  $a_{11} + a_{22} \neq 0$
- (B)  $a_{11}a_{22} + a_{12}a_{21} \neq 0$
- (C)  $a_{11}a_{22} - a_{12}a_{21} \neq 0$
- (D) All four entries  $a_{ij}$  are nonzero

(b) If the matrix is invertible, what is  $A^{-1}$ ? The (1, 1) entry of  $A^{-1}$  is

- (A)  $\frac{a_{11}}{a_{11}a_{22} - a_{12}a_{21}}$
- (B)  $\frac{a_{22}}{a_{11}a_{22} - a_{12}a_{21}}$
- (C)  $\frac{-a_{12}}{a_{11}a_{22} - a_{12}a_{21}}$
- (D)  $\frac{-a_{21}}{a_{11}a_{22} - a_{12}a_{21}}$

(c) Show that if the two columns of  $A$  are linearly dependent, the matrix is not invertible. If the columns are linearly dependent, then

- (A)  $A^{-1}$  exists but is not unique
- (B)  $a_{11}a_{22} - a_{12}a_{21} \neq 0$
- (C)  $a_{11}a_{22} - a_{12}a_{21} = 0$
- (D)  $a_{11}a_{22} + a_{12}a_{21} = 0$

**Exercise 3 (Least-squares estimation of activation energy<sup>1</sup>).**

Assume you have measured a rate constant,  $k$ , at several different temperatures,  $T$ , and wish to find the activation energy (divided by the gas constant),  $E/R$ , and the preexponential factor,  $k_0$ , in the Arrhenius model

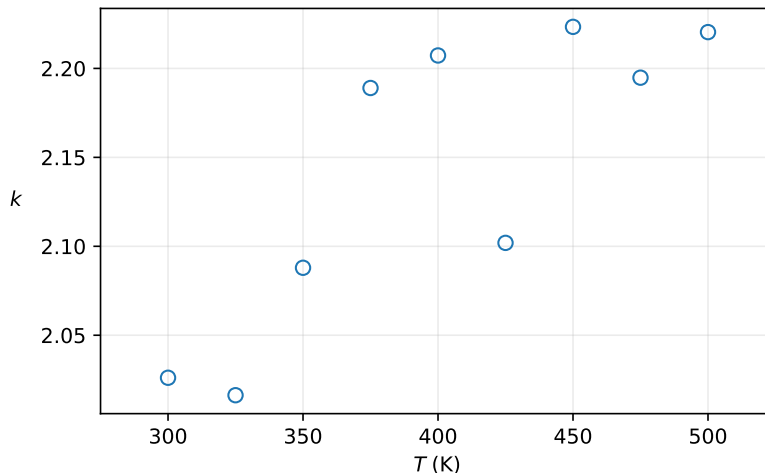
$$k = k_0 e^{-E/RT} \quad (1)$$

The data are shown in Figure 1 and listed here.

$T$ (K)	300	325	350	375	400	425	450	475	500
$k$	2.03	2.02	2.09	2.19	2.21	2.10	2.22	2.19	2.22

(a) Take logarithms of (1) and write a model linear in  $\ln(k_0)$  and  $E/R$ , then set up the matrix least-squares problem  $Ax = b$ . The rows of  $A$  are of the form

<sup>1</sup>See also (Graham and Rawlings, 2022, Exercise 1.22).



**Figure 1:** Measured rate constant at several temperatures.

- (A)  $[1 \quad T_i]$   
 (B)  $[\ln T_i \quad -1/T_i]$   
 (C)  $[T_i \quad -1]$   
 (D)  $[1 \quad -1/T_i]$
- (b) Find the least-squares solution  $\hat{x} = (A^T A)^{-1} A^T b$ . The estimates (to three significant figures) are
- (A)  $\ln(\hat{k}_0) = 0.154$ ,  $\hat{E}/R = 7.09$  K  
 (B)  $\ln(\hat{k}_0) = -0.943$ ,  $\hat{E}/R = 70.9$  K  
 (C)  $\ln(\hat{k}_0) = 0.943$ ,  $\hat{E}/R = 70.9$  K  
 (D)  $\ln(\hat{k}_0) = 0.943$ ,  $\hat{E}/R = -70.9$  K
- (c) Is the least-squares solution unique? The solution is unique because
- (A) The matrix  $A$  is square  
 (B) All entries of  $b$  are positive  
 (C) The matrix  $A^T A$  is diagonal  
 (D) The columns of  $A$  are linearly independent
- (d) Plot the data and least-squares fit in the original variables  $k$  versus  $T$ . The quality of the fit is
- (A) Poor; the model shows significant systematic deviations from the data  
 (B) Acceptable; but a nonlinear fit would be much better  
 (C) Good; the Arrhenius model captures the trend in the data  
 (D) Perfect; all data points lie exactly on the curve

## References

M. D. Graham and J. B. Rawlings. *Modeling and Analysis Principles for Chemical and Biological Engineers*. Nob Hill Publishing, Santa Barbara, CA, 2nd, paperback edition, 2022. 560 pages, ISBN 978-0-9759377-6-1.