Homework 2 — Multiple Choice

ChE 132A

Exercise 1 (More classifying differential equations).

(a) The equation $\dfrac{d^2u}{dx^2} + 4\dfrac{du}{dx} + 4u = 0$ is best described as

(A) ODE, first-order, linear, homogeneous
(B) ODE, second-order, linear, homogeneous
(C) PDE, second-order, linear
(D) ODE, second-order, nonlinear

(b) The equation $\dfrac{df}{dt} + f^2 = \sin t$ is

(A) First-order, linear, homogeneous
(B) First-order, nonlinear
(C) Second-order, linear, nonhomogeneous
(D) Second-order, nonlinear

(c) The equation $\dfrac{\partial c}{\partial t} = D\,\dfrac{\partial^2 c}{\partial x^2}$ is best described as

(A) ODE, second-order, linear, homogeneous
(B) PDE, second-order, linear, homogeneous
(C) ODE, first-order, nonlinear
(D) PDE, first-order, linear

(d) The equation $y'' + x\,y' + 2y = 0$ is best described as

(A) ODE, second-order, nonlinear
(B) ODE, second-order, linear, homogeneous
(C) ODE, first-order, linear, nonhomogeneous
(D) PDE, second-order, linear

(e) The equation $y'' + y = 3\cos x$ is nonhomogeneous because

(A) The coefficient of y is a constant rather than a function of x
(B) The right-hand side $3\cos x$ is not identically zero
(C) The order is greater than one
(D) Setting $y = 0$ satisfies the equation

(f) The equation $y\,\dfrac{dy}{dx} + x = 0$ is

(A) First-order, linear, solvable by an integrating factor

- (B) First-order, nonlinear, solvable by separation of variables
- (C) Second-order, nonlinear
- (D) First-order, nonlinear, and not separable

(g) For equations (a), (d), (e), and (f), identify which solution method from the handout applies: integrating factor, separation of variables, characteristic equation (constant-coefficient homogeneous), undetermined coefficients, or power series.

- (A) (a) integrating factor; (d) undetermined coefficients; (e) power series; (f) characteristic equation
- (B) (a) characteristic equation; (d) power series; (e) separation of variables; (f) integrating factor
- (C) (a) characteristic equation; (d) power series; (e) undetermined coefficients; (f) separation of variables
- (D) (a) separation of variables; (d) integrating factor; (e) characteristic equation; (f) undetermined coefficients

Exercise 3 (Constant-coefficient second-order equations).

(a) The characteristic roots of $u'' - u' - 6u = 0$ are

- (A) $\lambda_1 = 3, \lambda_2 = -2$
- (B) $\lambda_1 = -3, \lambda_2 = 2$
- (C) $\lambda_1 = 2, \lambda_2 = 3$
- (D) $\lambda_1 = 1, \lambda_2 = -6$

(b) For $u'' - u' - 6u = 0$ with $u(0) = 1$ and $u'(0) = 0$, the constants in $u = c_1 e^{3x} + c_2 e^{-2x}$ are

- (A) $c_1 = 1/2, c_2 = 1/2$
- (B) $c_1 = 3/5, c_2 = 2/5$
- (C) $c_1 = 2/5, c_2 = 3/5$
- (D) $c_1 = 1, c_2 = 0$

(c) The general solution of $u'' - 4u' + 13u = 0$ is

- (A) $c_1 e^{2x} + c_2 e^{3x}$
- (B) $e^{4x}(c_1 \cos 3x + c_2 \sin 3x)$
- (C) $e^{-2x}(c_1 \cos 3x + c_2 \sin 3x)$
- (D) $e^{2x}(c_1 \cos 3x + c_2 \sin 3x)$

Exercise 5 (Power-series solution).

(a) Written in standard form, $(x^2 - 9)\, y'' + 3x\, y' + y = 0$ has singular points at

- (A) $x = 0$ only

(B) $x = \pm 9$
(C) $x = 3$ only
(D) $x = \pm 3$

(b) The radius of convergence of a power-series solution of $(x^2 - 9)\,y'' + 3x\,y' + y = 0$ expanded about $x = 0$ is

(A) $R = 1$
(B) $R = 3$
(C) $R = 9$
(D) $R = \infty$

(c) Substituting $y = \sum_{n=0}^{\infty} c_n x^n$ into $y'' + xy' + y = 0$ and collecting powers of x^n gives the recurrence relation

(A) $c_{n+2} = -c_n/(n+1)$
(B) $c_{n+1} = -c_n/(n+2)$
(C) $c_{n+2} = c_n/(n+2)$
(D) $c_{n+2} = -c_n/(n+2)$

(d) For the even-series solution ($c_0 = 1$, $c_1 = 0$), the coefficient c_4 equals

(A) $-1/8$
(B) $1/4$
(C) $-1/4$
(D) $1/8$

Exercise 6 (Batch reactor kinetics).

A first-order irreversible reaction $A \longrightarrow B$ occurs in a batch reactor:

$$\frac{dc_A}{dt} = -kc_A$$

(a) Which type of differential equation is this?

(A) ODE, Nonlinear, second-order
(B) ODE, Linear, first-order, homogeneous
(C) ODE, Separable but not linear
(D) PDE

(b) The solution for $c_A(t)$ with $c_A(0) = c_{A0}$ is

(A) $c_A = c_{A0} + kt$
(B) $c_A = c_{A0}e^{-kt}$
(C) $c_A = \dfrac{1}{kt + C}$
(D) $c_A = e^{kt}$

References