Homework 3 — Multiple Choice

ChE 132A

Exercises

Exercise 1 (Differentiating integrals).

Taken from (Graham and Rawlings, 2022, Exercise 2.11).

(a) To solve the first-order linear ODE

$$\frac{dy}{dt} + p(t)\, y = q(t)$$

the standard technique is to multiply both sides by an integrating factor. Which of the following is the correct integrating factor?

(A) $e^{-\int_0^t p(t')\,dt'}$

(B) $e^{-p(t)}$

(C) $e^{\int_0^t p(t')\,dt'}$

(D) $p(t)e^t$

(b) After multiplying by the integrating factor and integrating, the solution contains terms like $\int_0^t q(t'')\, e^{\int_0^{t''} p(\tau)\,d\tau}\, dt''$ and an initial condition term y_0. When checking that the solution satisfies the initial condition $y(0) = y_0$, what happens to the integral term at $t = 0$?

(A) It equals y_0, so the initial condition is automatically satisfied

(B) It equals zero because both limits of integration are equal

(C) It diverges and needs to be regularized

(D) It contributes an unknown constant that must be determined separately

(c) For a double integral

$$f(t) = \int_{a(t)}^{b(t)} \int_{c(t,p)}^{d(t,p)} h(t, p, s)\ \mathrm{d}s\ \mathrm{d}p,$$

the Leibniz rule for $\frac{df}{dt}$ includes boundary terms from moving limits and interior terms from partial derivatives. Which of the following correctly lists all sources of contributions?

(A) Only the outer boundary terms: $b'(t)$ and $a'(t)$ times the inner integral

(B) Only the interior partial derivatives: $\int\int \frac{\partial h}{\partial t}\, ds\, dp$

(C) Outer boundaries ($b'(t)$, $a'(t)$), inner boundaries (d_t, c_t), and interior partial ($\partial h/\partial t$)

(D) Only the inner boundary terms from $d(t, p)$ and $c(t, p)$

Exercise 2 (Convolution theorem).

Taken from (Graham and Rawlings, 2022, Exercise 2.12).

(a) The convolution of two functions $f(t)$ and $g(t)$ is defined as

- (A) $\int_0^\infty f(t)\, g(t)\, dt$
- (B) $f(t) \cdot g(t)$ (pointwise multiplication)
- (C) $\int_0^t f(t')\, g(t-t')\, dt'$
- (D) $f(t) + g(t)$ (pointwise addition)

(b) Starting with the Laplace transform definition

$$\mathcal{L}\left\{\int_0^t f(t')\, g(t-t')\, dt'\right\} = \int_0^\infty e^{-st} \int_0^t f(t')\, g(t-t')\, dt' \; \mathrm{d}t,$$

the first step to derive the convolution theorem is to switch the order of integration. After switching, the region of integration can be rewritten as

- (A) $\int_0^t \int_0^\infty$ (same—order doesn't matter)
- (B) $\int_0^\infty \int_t^\infty$ with t' ranging from 0 to ∞ and then t from 0 to ∞
- (C) $\int_0^\infty \int_{t'}^\infty$ with t' fixed and then integrating t from t' to ∞
- (D) $\int_0^\infty \int_0^{t'}$ (reversed limits)

(c) After switching order, we have

$$\int_0^\infty \int_{t'}^\infty e^{-st} f(t')\, g(t-t') \; \mathrm{d}t \; \mathrm{d}t'.$$

Making the substitution $\tau = t - t'$ in the inner integral converts e^{-st} to

- (A) $e^{-s\tau}$ only
- (B) $e^{-st'}$ only
- (C) $e^{-s(\tau+t')}$, which factors as $e^{-s\tau}\, e^{-st'}$
- (D) e^{st} (the sign flips)

(d) After the substitution and factoring the exponentials, the double integral becomes a product of two separate integrals. This factorization gives

- (A) $\left[\int_0^\infty e^{-s\tau}\, d\tau\right] \left[\int_0^\infty e^{-st'}\, f(t')\, dt'\right]$
- (B) $\left[\int_0^\infty e^{-s\tau}\, g(\tau)\, d\tau\right] \left[\int_0^\infty e^{-st'}\, dt'\right]$
- (C) $\left[\int_0^\infty e^{-s\tau}\, g(\tau)\, d\tau\right] \left[\int_0^\infty e^{-st'}\, f(t')\, dt'\right]$
- (D) $\left[\int_0^\infty e^{-s\tau}\, d\tau\right] \left[\int_0^\infty e^{-st'}\, g(t')\, dt'\right]$

Exercise 3 (Final-value and initial-value theorems).

Taken from (Graham and Rawlings, 2022, Exercise 2.13).

(a) Which of the following transforms satisfies the conditions of the final-value theorem and therefore has a finite final value?

- (A) $\dfrac{1}{s^2}$

(B) $\dfrac{1}{s(s-a)}$ with $\text{Re}(a) > 0$

(C) $\dfrac{1}{s(s+a)}$ with $\text{Re}(a) > 0$

(D) $\dfrac{1}{s-1}$

(b) Using the initial-value theorem, $f(0^+) = \lim_{s\to\infty} s\,\overline{f}(s)$. For $\overline{f}(s) = 1/s^2$, the initial value is

(A) 1

(B) 0

(C) ∞

(D) does not exist

(c) If $\overline{f}(s) = 1/(s(s+a))$ with $\text{Re}(a) > 0$, then the final value is

(A) 0

(B) a

(C) $-a$

(D) $1/a$

References

M. D. Graham and J. B. Rawlings. *Modeling and Analysis Principles for Chemical and Biological Engineers*. Nob Hill Publishing, Santa Barbara, CA, 2nd, paperback edition, 2022. 560 pages, ISBN 978-0-9759377-6-1.