

Homework 4 — Multiple Choice

ChE 132A

Exercises

Exercise 6 (Differentiating a Laplace transform).

Taken from (Graham and Rawlings, 2022, Exercise 2.16).

Starting from the definition $\bar{f}(s) = \int_0^\infty e^{-st} f(t) dt$, differentiating with respect to s establishes Property 4 of the Laplace transform.

(a) Differentiating $\bar{f}(s)$ under the integral sign with respect to s gives

(A) $\int_0^\infty e^{-st} f'(t) dt$

(B) $\int_0^\infty t e^{-st} f(t) dt$

(C) $\int_0^\infty (-t) e^{-st} f(t) dt$

(D) $-\int_0^\infty e^{-st} f(t) dt$

(b) Rearranging shows that

(A) $\mathcal{L}\{tf(t)\} = \frac{d\bar{f}}{ds}$

(B) $\mathcal{L}\{tf(t)\} = -\frac{d\bar{f}}{ds}$

(C) $\mathcal{L}\{t^2 f(t)\} = -\frac{d\bar{f}}{ds}$

(D) $\mathcal{L}\{f(t)/t\} = -\frac{d\bar{f}}{ds}$

(c) Applying the same differentiation n times gives

(A) $\frac{d^n \bar{f}}{ds^n} = \mathcal{L}\{t^n f(t)\}$

(B) $\frac{d^n \bar{f}}{ds^n} = n! \bar{f}(s)$

(C) $\frac{d^n \bar{f}}{ds^n} = (-1)^n \mathcal{L}\{t^n f(t)\}$

(D) $\frac{d^n \bar{f}}{ds^n} = (-1)^n \bar{f}(s)$

Exercise 7 (Partial fraction expansion).

Taken from (Graham and Rawlings, 2022, Exercise 3.24).

For the partial fraction expansion of

$$\frac{1}{(s-a)(s-b)(s-c)} = \frac{A}{s-a} + \frac{B}{s-b} + \frac{C}{s-c}, \quad a \neq b \neq c$$

- (a) To find the coefficient A of the $\frac{1}{s-a}$ term, you multiply both sides by $(s-a)$ to obtain $1 = A(s-b)(s-c) + \dots$. Evaluating at $s = a$ eliminates the other terms. What do you get?

(A) $1 = A(a-b)(a-c)$

(B) $1 = A(b-a)(c-a)$

(C) $1 = A(a-b)(a-c)$

(D) $1 = A(a-b)(a-b)$

- (b) The standard inverse Laplace pair is $\mathcal{L}^{-1}\left\{\frac{1}{s-\alpha}\right\} = e^{\alpha t}$. Applying this to each term in the partial fraction expansion, $f(t)$ is

(A) $\frac{A}{s-a} + \frac{B}{s-b} + \frac{C}{s-c}$

(B) $A e^{-at} + B e^{-bt} + C e^{-ct}$

(C) $A e^{at} + B e^{bt} + C e^{ct}$

(D) $(A+B+C) e^{(a+b+c)t}$

- (c) When using the residue formula with $\bar{q}(s) = (s-a)(s-b)(s-c)$, the term containing $(s-a)$ vanishes at $s = a$, so $\bar{q}'(a)$ equals

(A) $a-b$

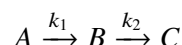
(B) $(a-b)(b-c)$

(C) $(a-b)(a-c)$

(D) $(a+b)(a+c)$

Exercise 16 (Two chemical reactions in series).

Consider the two irreversible reactions



taking place in a well-stirred batch reactor with rate expressions $r_1 = k_1 c_A$ and $r_2 = k_2 c_B$. Starting with only species A at $t = 0$, we have $c_{A0} > 0$ and $c_{B0} = c_{C0} = 0$.

- (a) Taking the Laplace transform of $\frac{dc_A}{dt} = -k_1 c_A$ with $c_A(0) = c_{A0}$ gives $s\bar{c}_A(s) - c_{A0} = -k_1 \bar{c}_A(s)$. Solving for $\bar{c}_A(s)$ yields

- (A) $\bar{c}_A(s) = \frac{c_{A0}k_1}{s}$
 (B) $\bar{c}_A(s) = \frac{1}{s+k_1}$
 (C) $\bar{c}_A(s) = \frac{s \cdot c_{A0}}{s+k_1}$
 (D) $\bar{c}_A(s) = \frac{c_{A0}}{s+k_1}$

(b) Taking the Laplace transform of $\frac{dc_B}{dt} = k_1c_A - k_2c_B$ with $c_B(0) = 0$ and substituting $\bar{c}_A(s) = \frac{c_{A0}}{s+k_1}$ gives $(s+k_2)\bar{c}_B(s) = k_1\bar{c}_A(s)$. The solution $\bar{c}_B(s)$ consists of

- (A) A single term from the forcing: $\frac{k_1c_{A0}}{(s+k_1)(s+k_2)}$
 (B) Only the homogeneous response: $\frac{0}{s+k_2}$
 (C) Two terms: the forced response plus a zero homogeneous response (since $c_{B0} = 0$)
 (D) A sum: $\frac{c_{B0}}{s+k_2} + \frac{k_1c_{A0}}{s+k_1}$

(c) Inverting the transform of A gives

- (A) $c_A(t) = c_{A0}e^{-k_1t}$
 (B) $c_A(t) = c_{A0}(1 - e^{-k_1t})$
 (C) $c_A(t) = e^{-k_1t}$
 (D) $c_A(t) = \frac{c_{A0}}{k_1}(1 - e^{-k_1t})$

(d) When inverting $\bar{c}_B(s)$, the term $\frac{k_1c_{A0}}{(s+k_1)(s+k_2)}$ can be split using partial fractions as (assuming $k_1 \neq k_2$)

- (A) $\frac{k_1c_{A0}}{k_1 - k_2} \left(\frac{1}{s+k_1} + \frac{1}{s+k_2} \right)$
 (B) $\frac{k_1c_{A0}}{(s+k_1)^2}$
 (C) $\frac{k_1c_{A0}}{k_2 - k_1} \left(\frac{1}{s+k_1} - \frac{1}{s+k_2} \right)$
 (D) $\frac{c_{A0}}{s+k_1} - \frac{c_{A0}}{s+k_2}$

(e) Summing all three mass balances gives $\frac{d}{dt}(c_A + c_B + c_C) = 0$, which means

- (A) $c_A + c_B + c_C = 0$ always
 (B) $c_A + c_B + c_C = c_{A0}$ (conservation of mass)
 (C) $c_A + c_B + c_C$ increases linearly with time
 (D) $c_A + c_B + c_C$ depends on the rate constants

References

M. D. Graham and J. B. Rawlings. *Modeling and Analysis Principles for Chemical and Biological Engineers*. Nob Hill Publishing, Santa Barbara, CA, 2nd, paperback edition, 2022. 560 pages, ISBN 978-0-9759377-6-1.