Cooperation-based optimization of industrial supply chains

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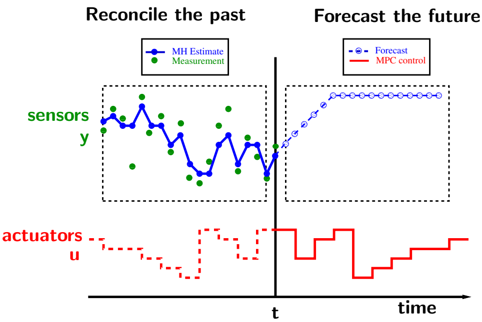
Lund University

Outline

1. Overview of Distributed Model Predictive Control
   - Control of large-scale systems
   - Stability theory for cooperative MPC
2. Challenges for Cooperative MPC of Supply Chains
3. Conclusions and Future Outlook

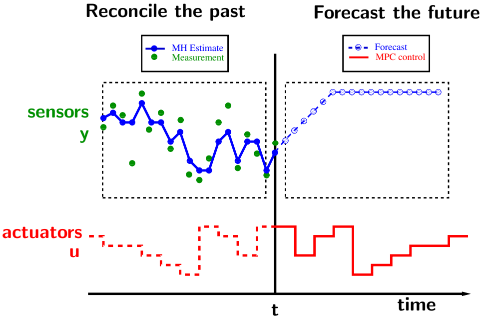
Predictive control



$$\min_{u(t)} \int_0^T |y_{sp} - g(x,u)|_Q^2 + |u_{sp} - u|_R^2 \, dt$$

$$\dot{x} = f(x,u)$$
$$x(0) = x_0 \quad \text{(given)}$$
$$y = g(x,u)$$

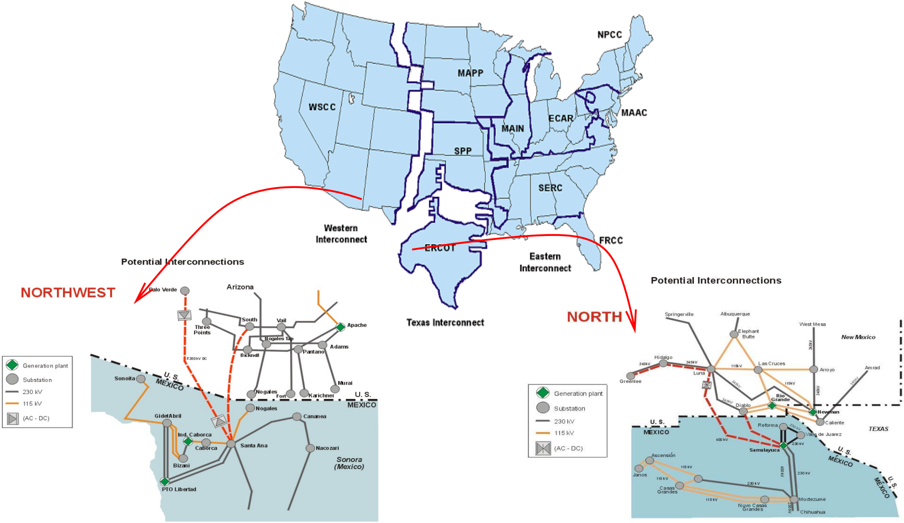
State estimation



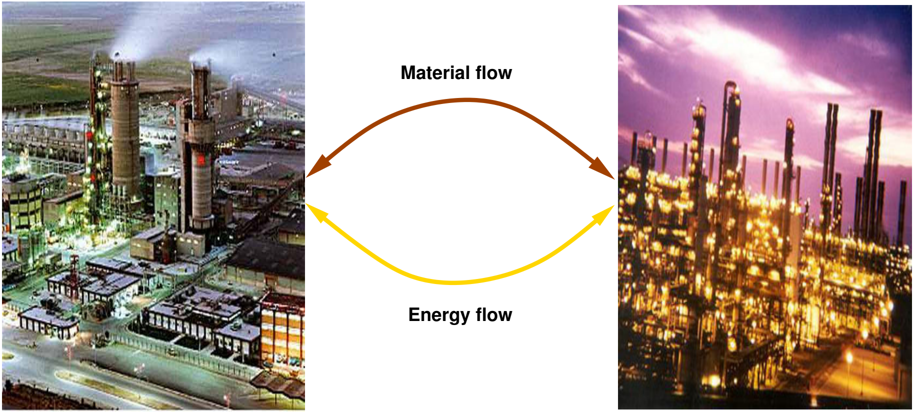
$$\min_{x_0, w(t)} \int_{-T}^{0} |y - g(x,u)|_R^2 + |\dot{x} - f(x,u)|_Q^2 \, dt$$

$$\begin{aligned} \dot{x} &= f(x,u) + w \quad &\text{(process noise)} \\ y &= g(x,u) + v \quad &\text{(measurement noise)} \end{aligned}$$

Electrical power distribution



Chemical plant integration



MPC at the large scale

Decentralized Control

- Most large-scale systems consist of networks of interconnected/interacting subsystems
  - Chemical plants, electrical power grids, water distribution networks, ...
- Traditional approach: Decentralized control
  - Wealth of literature from the early 1970's on improved decentralized control [a]
  - Well known that poor performance may result if the interconnections are not negligible

[a](Sandell Jr. et al., 1978; Šiljak, 1991; Lunze, 1992)

MPC at the large scale

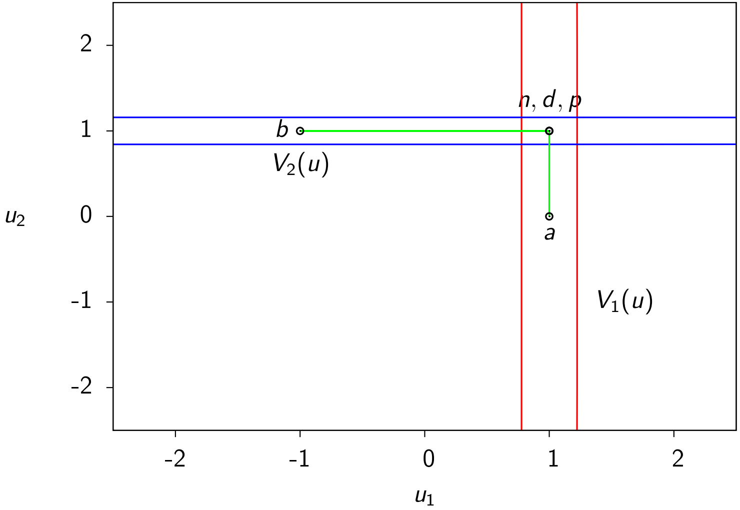
Centralized Control

- Steady increase in available computing power has provided the opportunity for centralized control
- Coordinated control: Distributed optimization to achieve fast solution of centralized control (Necoara et al., 2008; Cheng et al., 2007)
- Most practitioners view centralized control of large, networked systems as impractical and unrealistic
- A divide and conquer strategy is essential for control of large, networked systems (Ho, 2005)
- Centralized control: A benchmark for comparing and assessing distributed controllers

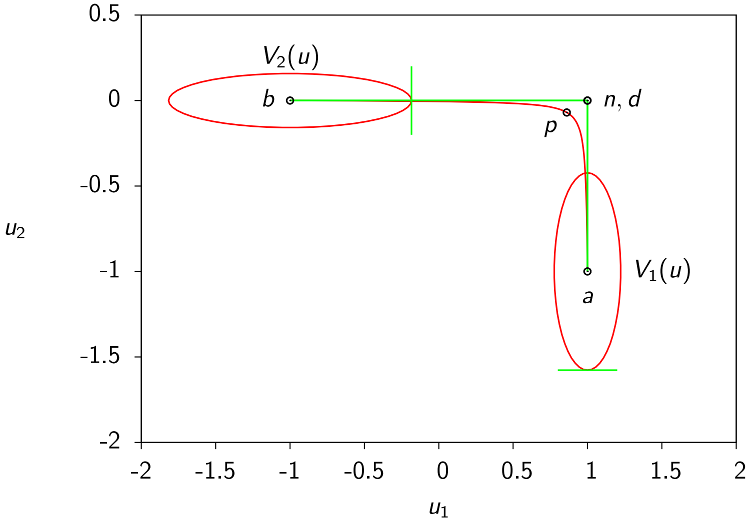
Nomenclature: consider two interacting units

Objective functions	$V_1(u_1, u_2), \; V_2(u_1, u_2)$
and	$V(u_1, u_2) = w_1 V_1(u_1, u_2) + w_2 V_2(u_1, u_2)$
decision variables for units	$u_1 \in \Omega_1, \quad u_2 \in \Omega_2$
Decentralized Control	$\min_{u_1 \in \Omega_1} \widetilde{V}_1(u_1) \quad \min_{u_2 \in \Omega_2} \widetilde{V}_2(u_2)$
Noncooperative Control	$\min_{u_1 \in \Omega_1} V_1(u_1, u_2) \quad \min_{u_2 \in \Omega_2} V_2(u_1, u_2)$
(Nash equilibrium)	
Cooperative Control	$\min_{u_1 \in \Omega_1} V(u_1, u_2) \quad \min_{u_2 \in \Omega_2} V(u_1, u_2)$
(Pareto optimal)	
Centralized Control	$\min_{u_1, u_2 \in \Omega_1 \times \Omega_2} V(u_1, u_2)$
(Pareto optimal)	

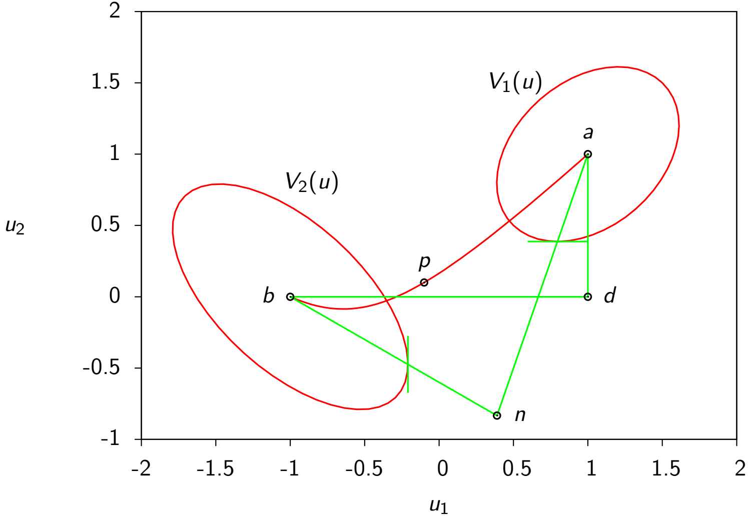
Noninteracting systems



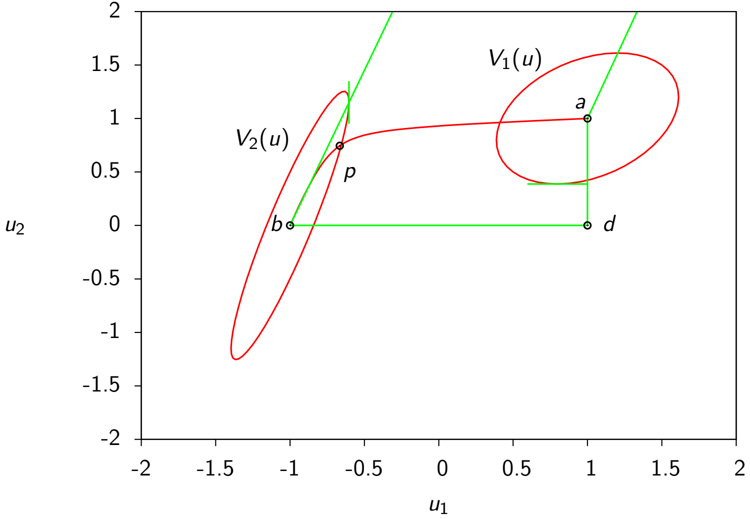
Weakly interacting systems



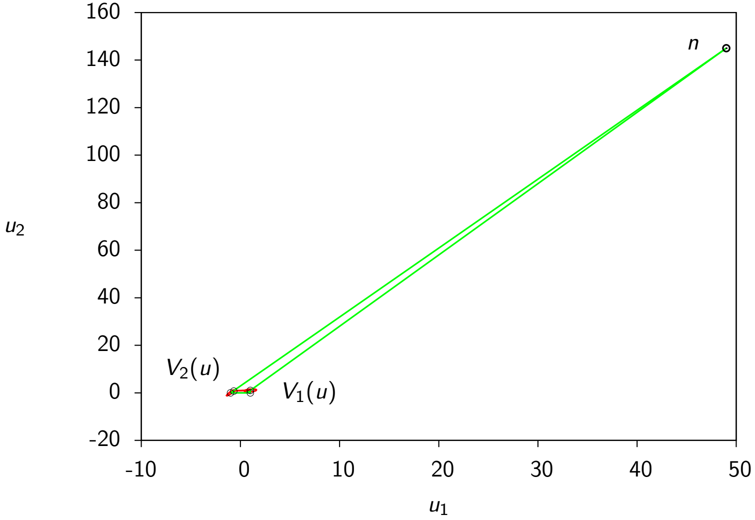
Moderately interacting systems



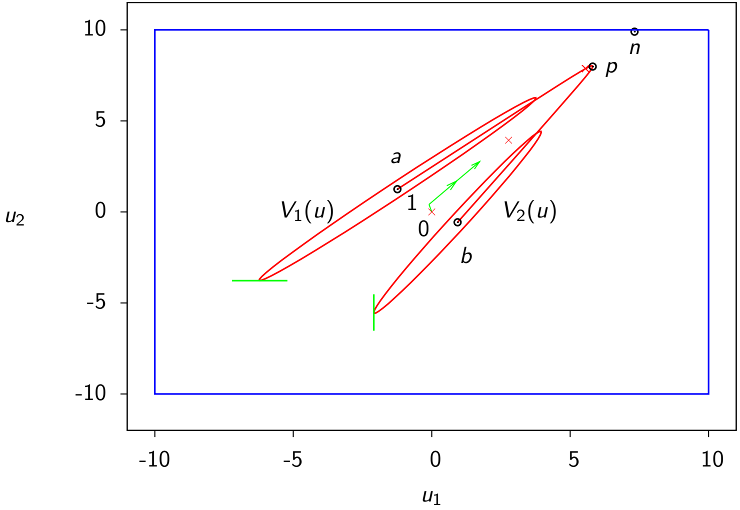
Strongly interacting (conflicting) systems



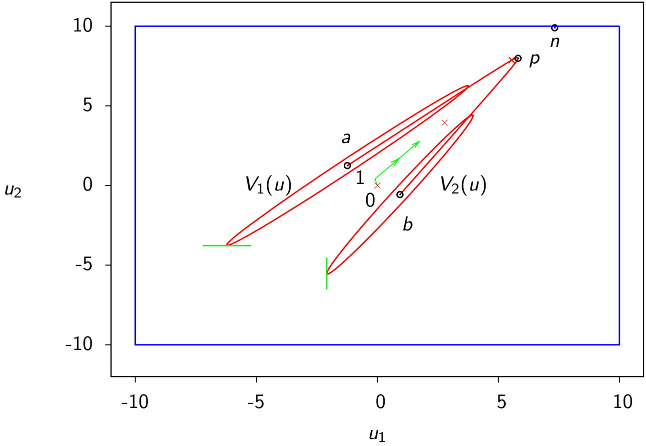
Strongly interacting (conflicting) systems



Geometry of cooperative vs. noncooperative MPC



Plantwide suboptimal MPC



- Early termination of optimization gives suboptimal plantwide feedback
- Use suboptimal MPC theory to prove stability

Plantwide suboptimal MPC

Consider closed-loop system augmented with input trajectory

$$\begin{pmatrix} x^+ \\ \mathbf{u}^+ \end{pmatrix} = \begin{pmatrix} Ax + Bu \\ g(x, \mathbf{u}) \end{pmatrix}$$

- Function $g(\cdot)$ returns suboptimal choice
- Stability of augmented system is established by Lyapunov function

$$a\left|(x, \mathbf{u})\right|^2 \leq V(x, \mathbf{u}) \leq b\left|(x, \mathbf{u})\right|^2$$
$$V(x^+, \mathbf{u}^+) - V(x, \mathbf{u}) \leq -c\left|(x, u)\right|^2$$

- Adding constraint establishes closed-loop stability of the origin for all $\mathbf{u}$[1]

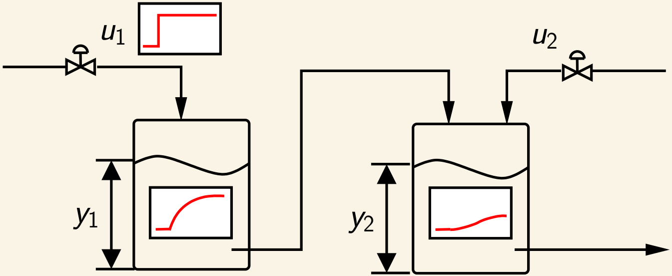
$$|\mathbf{u}| \leq d\,|x| \quad x \in \mathbb{B}_r, r > 0$$

- Cooperative optimization satisfies these properties for plantwide objective function $V(x, \mathbf{u})$

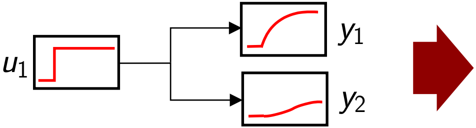
[1](Rawlings and Mayne, 2009, pp.418-420)

Modeling

Plantwide step response



- Interaction models found by decentralized identification[2]



$$x_{11}^+ = A_{11}x_{11} + B_{11}u_1$$

$$x_{21}^+ = A_{21}x_{21} + B_{21}u_1$$

[2]Gudi and Rawlings (2006)

Modeling

Consider the linearized physical model

$$x^+ = Ax + B_1 u_1 + B_2 u_2 \qquad y_1 = C_1 x, \quad y_2 = C_2 x$$

- Kalman canonical form of the triple (A, B_j, C_i)

$$\begin{bmatrix} z_{ij}^{oc} \\ z_{ij}^{\bar{o}c} \\ z_{ij}^{o\bar{c}} \\ z_{ij}^{\bar{o}\bar{c}} \end{bmatrix}^+ = \begin{bmatrix} A_{ij}^{oc} & 0 & A_{ij}^{oc\bar{c}} & 0 \\ A_{ij}^{\bar{o}oc} & A_{ij}^{\bar{o}c} & A_{ij}^{\bar{o}co\bar{c}} & A_{ij}^{\bar{o}c\bar{c}} \\ 0 & 0 & A_{ij}^{o\bar{c}} & 0 \\ 0 & 0 & A_{ij}^{\bar{o}\bar{c}o} & A_{ij}^{\bar{o}\bar{c}} \end{bmatrix} \begin{bmatrix} z_{ij}^{oc} \\ z_{ij}^{\bar{o}c} \\ z_{ij}^{o\bar{c}} \\ z_{ij}^{\bar{o}\bar{c}} \end{bmatrix} + \begin{bmatrix} B_{ij}^{oc} \\ B_{ij}^{\bar{o}c} \\ 0 \\ 0 \end{bmatrix} u_j$$

$$y_{ij} = \begin{bmatrix} C_{ij}^{oc} & 0 & C_{ij}^{o\bar{c}} & 0 \end{bmatrix} \begin{bmatrix} z_{ij}^{oc} \\ z_{ij}^{\bar{o}c} \\ z_{ij}^{o\bar{c}} \\ z_{ij}^{\bar{o}\bar{c}} \end{bmatrix} \qquad y_i = \sum_j y_{ij}$$

- Interaction models

$$A_{ij} \leftarrow A_{ij}^{oc} \quad B_{ij} \leftarrow B_{ij}^{oc} \quad C_{ij} \leftarrow C_{ij}^{oc} \quad x_{ij} \leftarrow z_{ij}^{oc}$$

Unstable modes

For unstable systems, we zero the unstable modes with terminal constraints.

- For subsystem 1

$$S_{11}^{u}{}' x_{11}(N) = 0 \qquad S_{21}^{u}{}' x_{21}(N) = 0$$

- To ensure terminal constraint feasibility for all x, we require $(\underline{A}_1, \underline{B}_1)$ stabilizable

$$\underline{A}_1 = \begin{bmatrix} A_{11} & \\ & A_{21} \end{bmatrix} \qquad \underline{B}_1 = \begin{bmatrix} B_{11} \\ B_{21} \end{bmatrix}$$

- For output feedback, we require (A_1, C_1) detectable

$$A_1 = \begin{bmatrix} A_{11} & \\ & A_{12} \end{bmatrix} \quad C_1 = \begin{bmatrix} C_{11} & C_{12} \end{bmatrix}$$

- Similar requirements for other subsystem

Output feedback

Consider augmented system perturbed by stable estimator

$$\begin{pmatrix} \hat{x}^+ \\ \mathbf{u}^+ \\ e^+ \end{pmatrix} = \begin{pmatrix} A\hat{x} + B\mathbf{u} + Le \\ g(\hat{x}, \mathbf{u}, e) \\ A_L e \end{pmatrix}$$

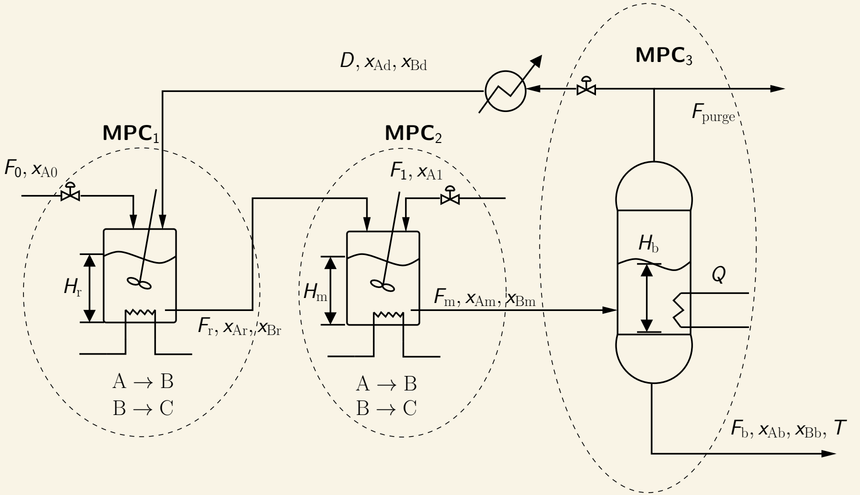
- Stable estimator error implies Lyapunov function

$$\begin{aligned} \bar{a}\,|e| \leq & J(e) \leq \bar{b}\,|e| \\ J(e^+) - & J(e) \leq -\bar{c}\,|e| \end{aligned}$$

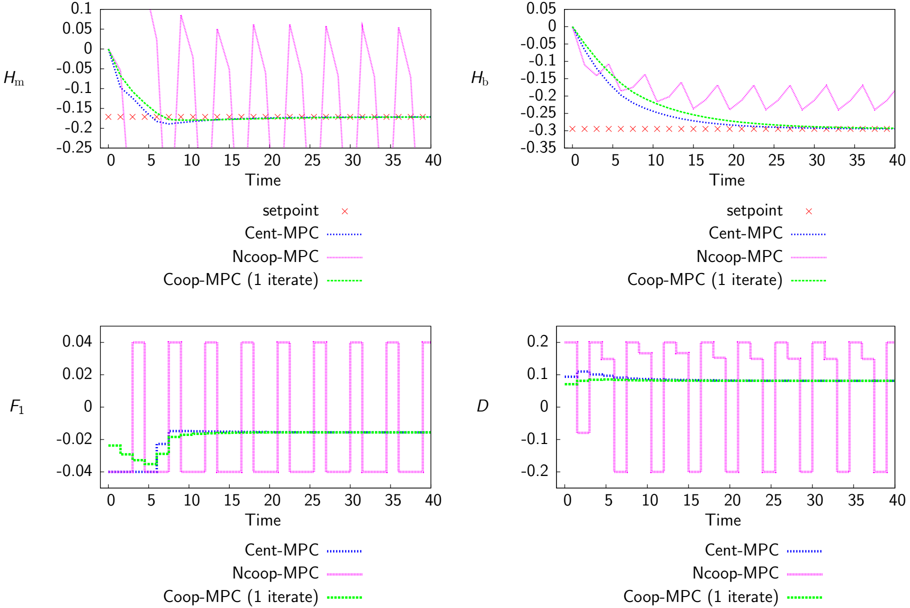
- Stability of perturbed system established by Lyapunov function

$$W(\hat{x}, \mathbf{u}, e) = V(\hat{x}, \mathbf{u}) + J(e)$$

Two reactors with separation and recycle



Two reactors with separation and recycle



Two reactors with separation and recycle

Performance comparison

	Cost ($\times 10^{-2}$)	Performance loss
Centralized MPC	1.75	0
Decentralized MPC	∞	∞
Noncooperative MPC	∞	∞
Cooperative MPC (1 iterate)	2.2	25.7%
Cooperative MPC (10 iterates)	1.84	5%

Cooperative MPC of supply chains

- Previous work on supply chain modeling and optimization[3]
- Inventories and backorders are subsystem states
- Downstream product shipments and upstream orders are subsystem inputs
- Inventories and backorders modeled as integrators (tanks)
- Stabilizability and detectability assumptions not satisfied

$$\underline{\mathrm{A}}_i = \begin{bmatrix} I & \\ & I \end{bmatrix} \qquad \underline{\mathrm{B}}_i = \begin{bmatrix} B_{1i} \\ B_{2i} \end{bmatrix}$$

$$A_i = \begin{bmatrix} I & \\ & I \end{bmatrix} \qquad C_i = \begin{bmatrix} C_{i1} & C_{i2} \end{bmatrix}$$

- Implementation of cooperative MPC for supply chains remains a challenge

[3] Perea López et al. (2003); Mestan et al. (2006); Braun et al. (2002); Seferlis and Giannelos (2004)

Cooperative MPC of supply chains

Possible solution I: Coupled constraints

- Work with minimal (A, B, C) supply chain model
- Terminal constraint $S^{u\prime} x(N) = 0$ coupled in subsystem inputs

Challenge

- Cooperative optimization does not converge to Pareto optimum with coupled constraints
- Share coupled inputs among subsystems to achieve Pareto optimal performance
- In the limit of full supply chain coupling, each subsystem solves the centralized optimization

Alternative

- To avoid centralized optimization, share inputs with only nearest neighbors for near optimal performance

Cooperative MPC of supply chains

Possible solution II: Centralized estimation

- $(\underline{A}_i, \underline{B}_i)$ not stabilizable, but there is a stabilizable subspace $\underline{\mathbb{X}}_i$

$$\underline{\mathbb{X}}_i = \left\{ \underline{x}_i \mid \exists\ \mathbf{u}_i : \begin{bmatrix} \underline{A}_i^{n-1}\underline{B}_i & \cdots & \underline{B}_i \end{bmatrix} \mathbf{u}_i = -\underline{A}_i^n \underline{x}_i \right\}$$

- Any $\underline{x}_i \in \underline{\mathbb{X}}_i$ can be brought to the origin

Challenge

- Must ensure estimated states are in stabilizable subspace
- Estimation must be centralized

Trade-offs

- No coupled constraints, therefore cooperative optimization converges to Pareto optimum
- Easy to enforce $\underline{x}_i \in \underline{\mathbb{X}}_i$
- Subsystems must share output measurements
- Supply chain subsystems cannot choose estimators independently

Conclusions

- Cooperative MPC theory maturing[a]
  - satisfies hard input constraints
  - provides nominal stability for plants with even strongly interacting subsystems
  - retains closed-loop stability for early iteration termination
  - converges to Pareto optimal control in the limit of iteration
  - remains stable under perturbation from stable state estimator
  - avoids coordination layer
- Cooperative MPC for supply chains remains a challenge
  - stabilizability and detectability assumptions not satisfied
  - many alternative solution strategies exist
  - each strategy has drawbacks

[a] **?**Maestre et al. (2010)

Future directions

Supply chains

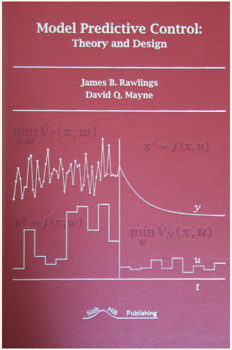
- Evaluate alternative supply chain cooperative control strategies
- Industrial application: gas supplier (Praxair), steel mill, power utility

Cooperative MPC

- Hierarchical implementation[a]
  - time scale separation
  - delayed communication
  - reduced information sharing
  - optimization at MPC layer only
- Nonlinear models

[a] Stewart et al. (2010)

MPC Monograph — Chapter 6 on distributed MPC



- 576 page text
- 214 exercises
- 335 page solution manual
- 3 appendices on web (133 pages)
- www.nobhillpublishing.com

Further reading I

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Further reading II

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