Economic optimization in Model Predictive Control

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Outline

Incentives for process control

- Production specifications

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- Operational constraints / Environmental regulations

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- Safety

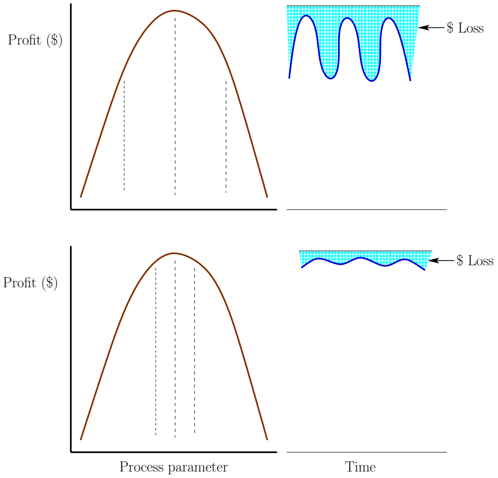
Incentives for process control

- Production specifications
- Operational constraints / Environmental regulations
- Safety
- **Economics**

Economic Incentive

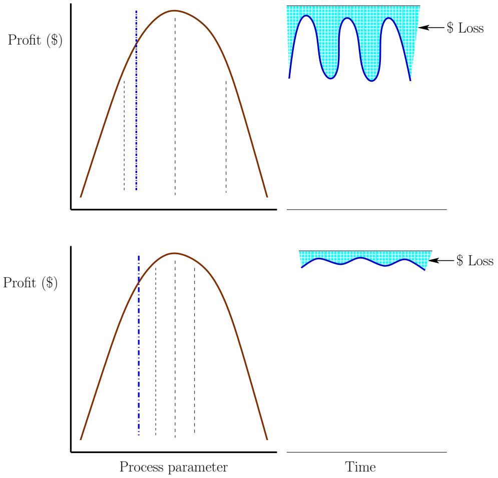
- Production of a plant depends heavily on plant's limitations and operating constraints
- Operating conditions keep changing plant production
- Under all variations and restrictions, plant must do the best it can: **Process optimization**

Global production maximum



- Higher profit expected when band of variation is reduced
- Allows operation at/near the optimum for more time
- Smoother operation $\Longrightarrow$ Higher profit

Global production maximum



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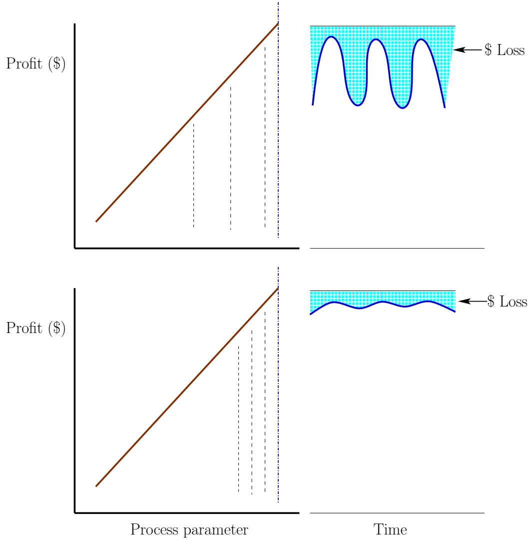
Maximum production at bound



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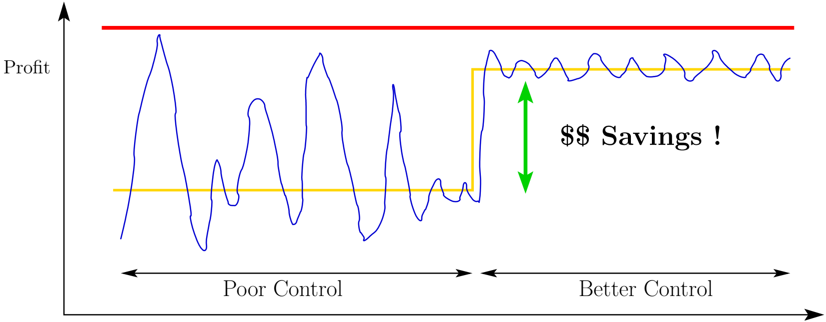
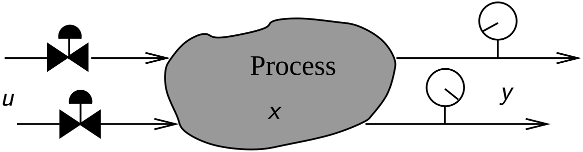
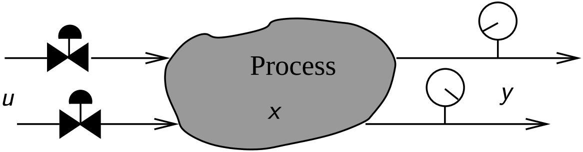


- Higher fluctuations: Poor disturbance rejection
- Forces the mean operating state to be away from optimum to meet the constraints
- **Solution**: Reduce fluctuations and go nearer to the optimal

Process model



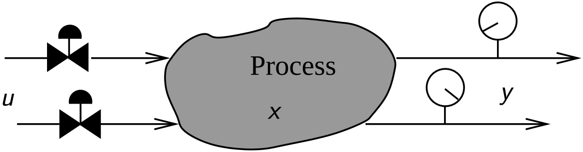
Process model



- Process model governing process dynamics

$$\frac{dx}{dt} = f(x(t), u(t))$$
$$y(t) = g(x(t))$$

Process model



- Process model governing process dynamics

$$\begin{aligned} \frac{dx}{dt} &= f(x(t), u(t)) \\ y(t) &= g(x(t)) \end{aligned}$$

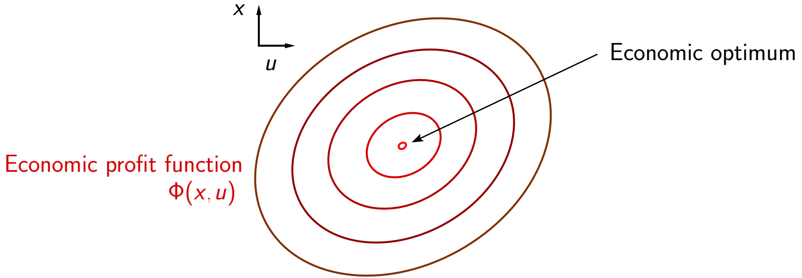
- Steady state:

$$\begin{aligned} f(x_s, u_s) &= 0 \\ y_s &= g(x_s) \end{aligned}$$

Objective translation

Economic objectives are translated into process control objectives

- Notion of setpoints / targets



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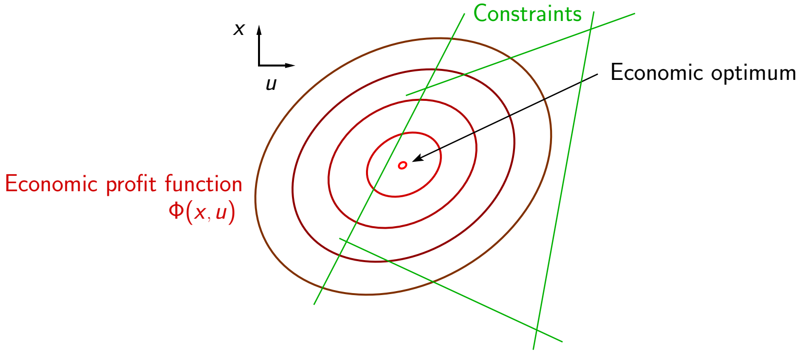

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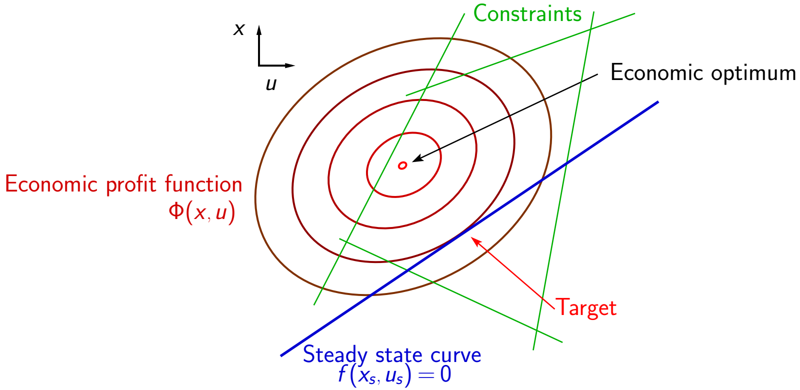

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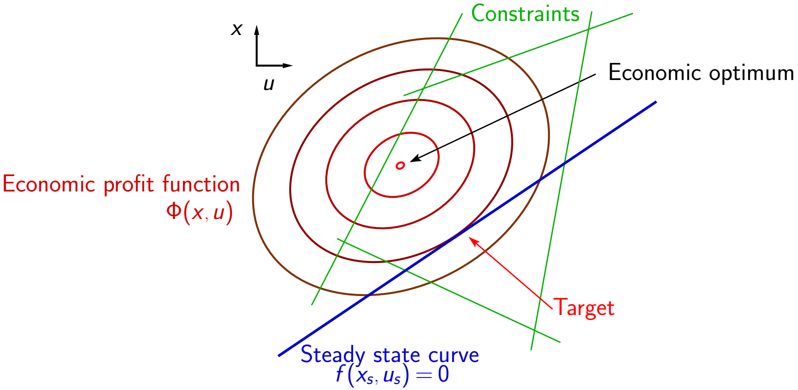
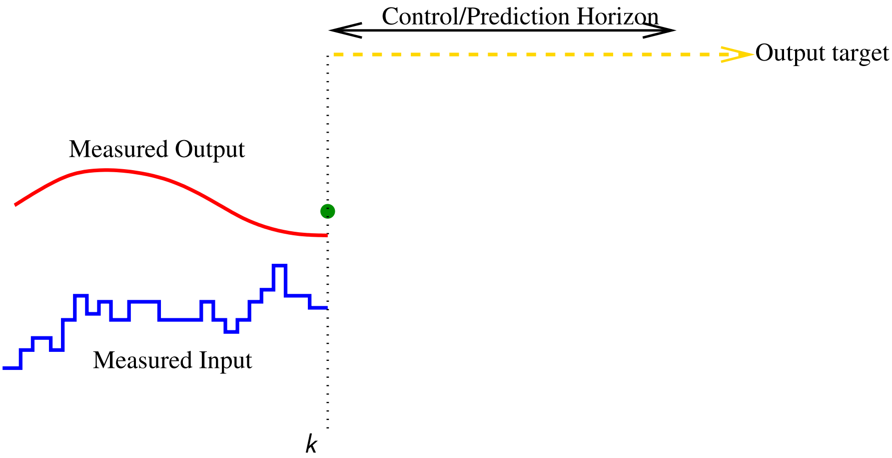
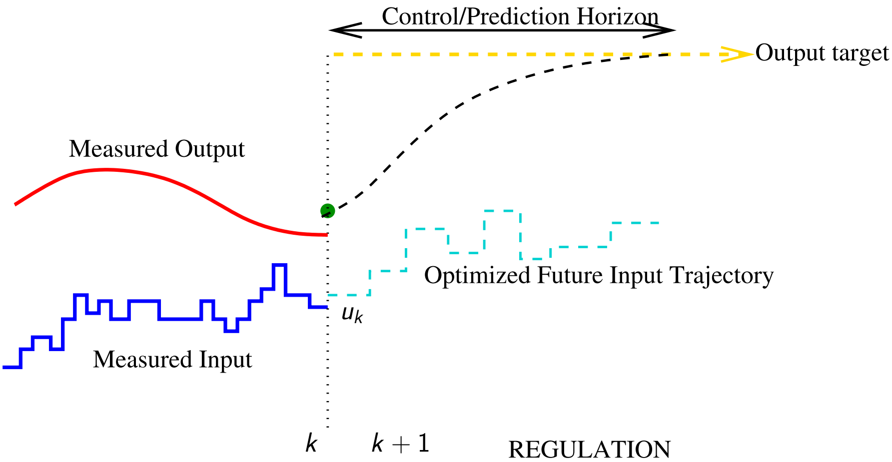


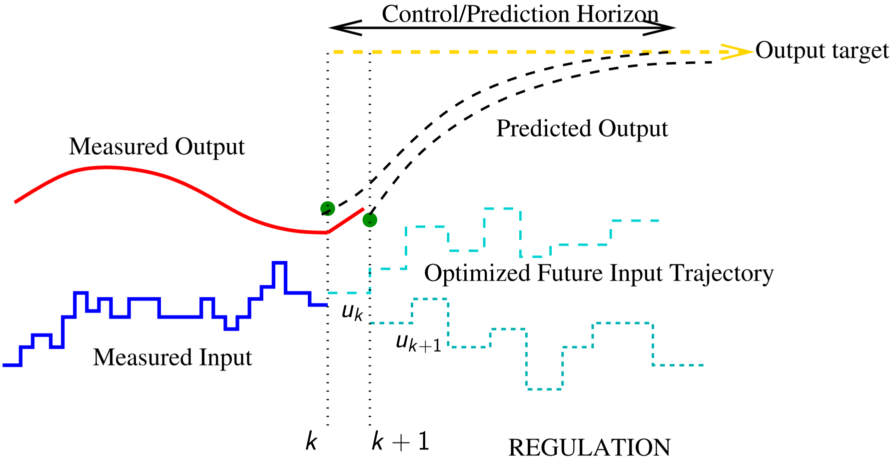
Model Predictive Control



Model Predictive Control



Model Predictive Control



Problem definition

- Get to the steady economic optimum (target): Minimize the distance from the target (stage cost)

$$L(\mathbf{x}, \mathbf{u}) = (x - x_t)' Q (x - x_t) + (u - u_t)' R (u - u_t)$$

Problem definition

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$$L(\mathbf{x}, \mathbf{u}) = (x - x_t)' Q (x - x_t) + (u - u_t)' R (u - u_t)$$

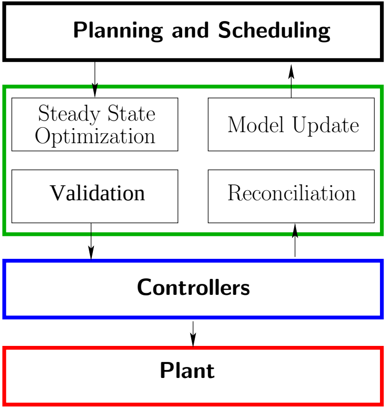
- Minimize the stage cost summed over a chosen control horizon (number of moves into the future: N)

$$\min_{\mathbf{u}} \sum_{i=0}^{N-1} L(\mathbf{x}, \mathbf{u})$$

- subject to the process model

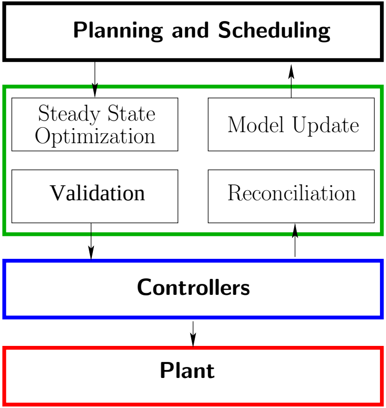
$$\mathbf{x}_{k+1} = \mathbf{A}\mathbf{x}_k + \mathbf{B}\mathbf{u}_k$$

Current practice: RTO



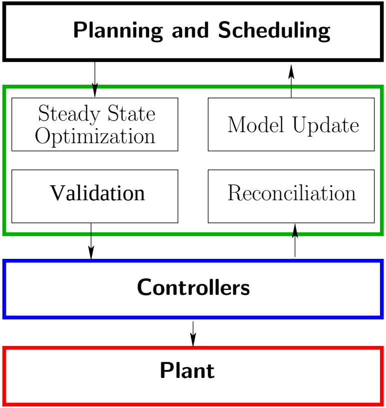
- Real time optimization

Current practice: RTO



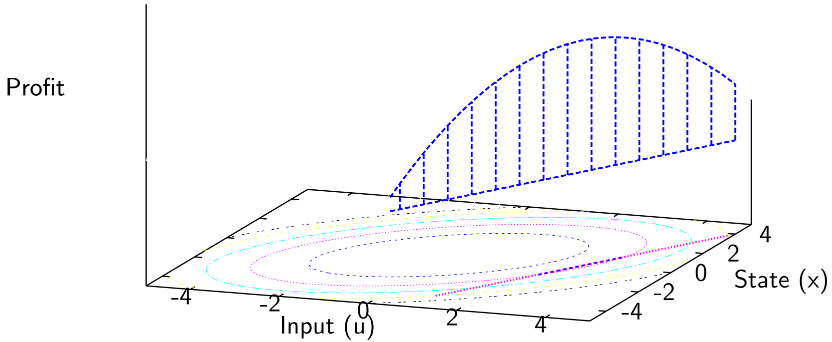
- Real time optimization
  - Two layer structure used to address economically optimal solution
  - RTO generated setpoints passed to lower level controller
  - Controllers try to "*track*" the targets provided to it

Current practice: RTO



- Real time optimization
  - Two layer structure used to address economically optimal solution
  - RTO generated setpoints passed to lower level controller
  - Controllers try to "*track*" the targets provided to it
- Drawbacks
  - Lower sampling rate
  - Adaptation of operating conditions is slow
  - *Consequence*: **Loss in economics**

Motivating the idea



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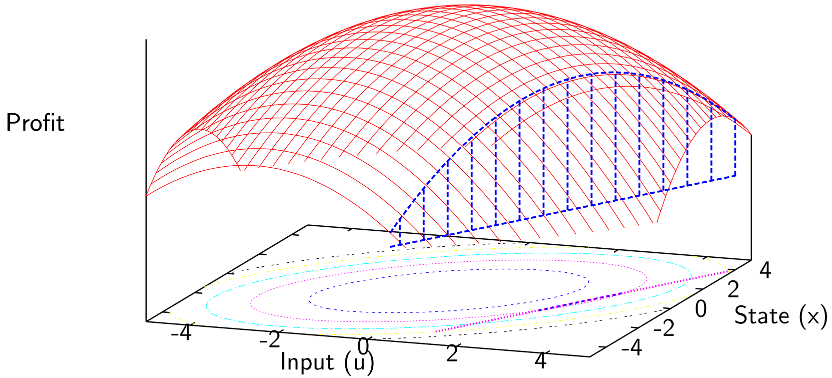

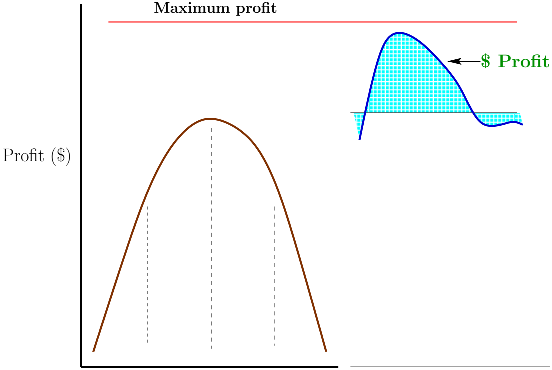

- Global economic optimum not being a steady state introduces high potential areas of transient operation
- Translation of economic objective to control objective loses the information about maximum profit possible

Motivating the idea

- What is **not** the primary objective of feedback control

Motivating the idea

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  - Tracking setpoints or targets
  - Tracking dynamic setpoint changes

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Process Economics

Steady state economics

Loss of economic information due to two layer approach

Control objective:

$$\mathbf{L}(\mathbf{x},\mathbf{u}) = (\mathbf{x}-\mathbf{x_t})'\mathbf{Q}(\mathbf{x}-\mathbf{x_t}) + (\mathbf{u}-\mathbf{u_t})'\mathbf{R}(\mathbf{u}-\mathbf{u_t})$$

Motivating the idea

- What is **not** the primary objective of feedback control
  - Tracking setpoints or targets
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Process Economics

Loss of economic information due to two layer approach

Control objective:

$$\mathbf{L}(\mathbf{x},\mathbf{u}) = -\mathbf{P}(\mathbf{x},\mathbf{u})$$

Make money or chase target ?

- Due to disturbances and constraints, the economic optimum is not a steady state in general
- System stabilizes at the steady target estimated from the steady state optimization
- During system transients, system may or may not pass through the economic optimum

The contest

- *The closer the system gets to the economic optimum, the more profitable it is*
- Who gets closest to the global economic optimum ?

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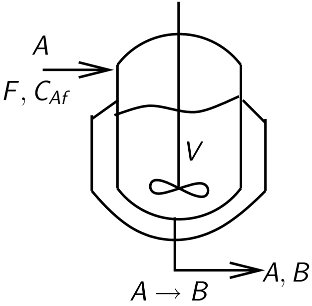
  - Tracking speed chosen through penalties, but still the objective remains to drive away from non steady economic optimum !

The contest

- *The closer the system gets to the economic optimum, the more profitable it is*
- Who gets closest to the global economic optimum ?
  - Tracking controllers: Rush to the target (*away from non steady economic optimum*)
  - Tracking speed chosen through penalties, but still the objective remains to drive away from non steady economic optimum !
- Economics optimizing controller: Expected to get closer to the optimum with eventual setting at the steady target

A motivating formulation

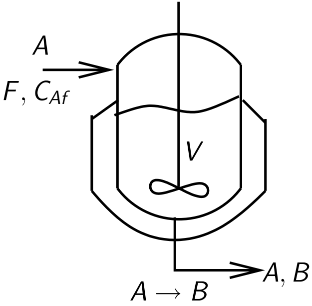


Consider a CSTR

$$V\frac{dC_A}{dt} = F(C_{Af} - C_A) - kC_A V$$
$$V\frac{dC_B}{dt} = F(C_{Bf} - FC_B) + kC_A V$$

- States: C_A, C_B Input: F

A motivating formulation



Consider a CSTR

$$V\frac{dC_A}{dt} = F(C_{Af} - C_A) - kC_AV$$
$$V\frac{dC_B}{dt} = F(C_{Bf} - FC_B) + kC_AV$$

- States: C_A, C_B Input: F

The simplest form of profit:

$$\begin{aligned} P &= \alpha_A F(C_A - C_{Af}) + \alpha_B F(C_B) \\ &= \begin{bmatrix} \mathbf{C_A} & \mathbf{C_B} \end{bmatrix} \begin{bmatrix} \alpha_\mathbf{A} \\ \alpha_\mathbf{B} \end{bmatrix}' \mathbf{F} - \alpha_A C_{Af} F \end{aligned}$$

α_A: Cost of A α_B: Cost of B

State-Input Cross term !

Example: Single input single output

- Consider a linear system

$$x_{k+1} = 0.3x_k + u_k$$

- Profit function: $-3x_k^2 - 5u_k^2 \mathbf{-2x_k u_k} + 98x_k + 80u_k$
- Objective: Maximize Profit !

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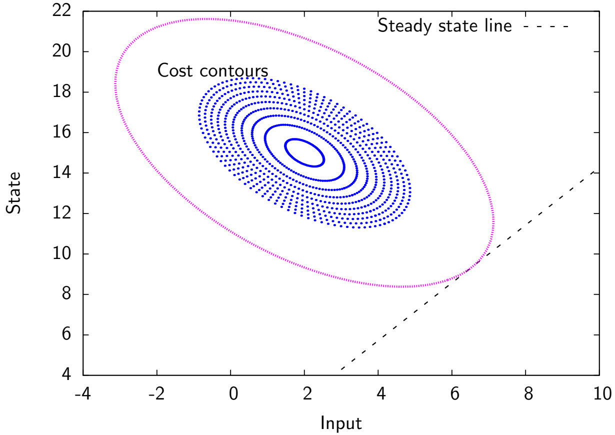
- **Scheme one**:
  - Evaluate the best economic target at every sample time (RTO)
  - Controller tracks the target given to it


Steady state line
Cost contours
State
Input
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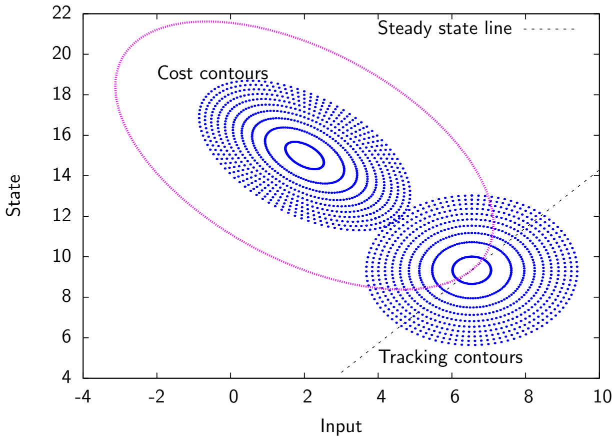
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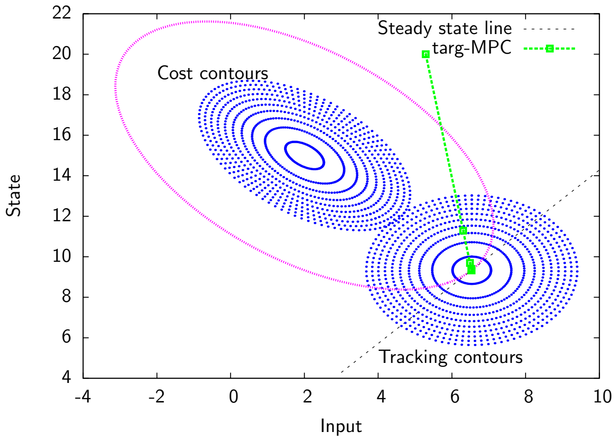
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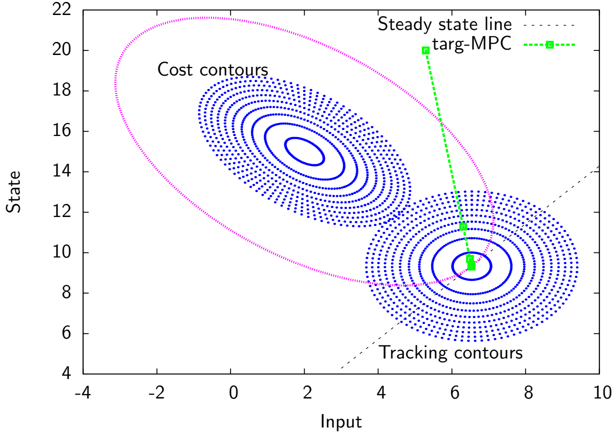
State
Input
Steady state line
targ-MPC
Cost contours
Tracking contours

- Profit function: $-3x_k^2 - 5u_k^2 - \mathbf{2x_k u_k} + 98x_k + 80u_k$
- Objective: Maximize Profit !
- **Scheme two**:
  - Controller minimizes the negative of profit


Steady state line
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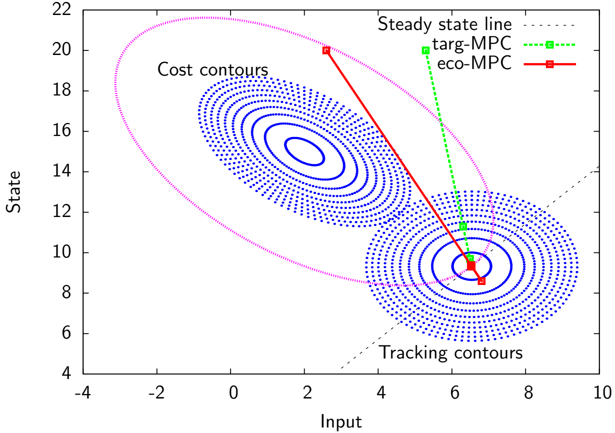
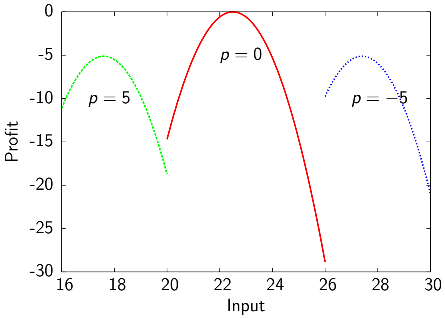


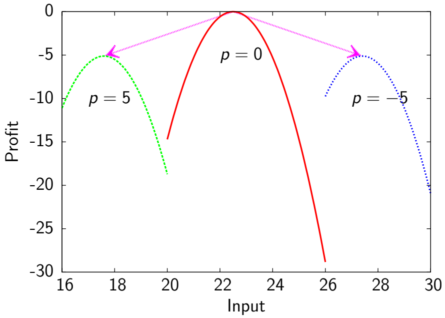
Performance Measures

	targ-MPC	eco-MPC	Δ(index)%
Loss[a]	\$642.6	\$588.2	8.5

[a] Reference: Maximum profit = 0



- Disturbance model:
  $x_{k+1} = Ax_k + Bu_k + B_d p_k$
- Disturbance shifts the steady state cost curve
- The steady state target changes
- System transients from previous target to the new target



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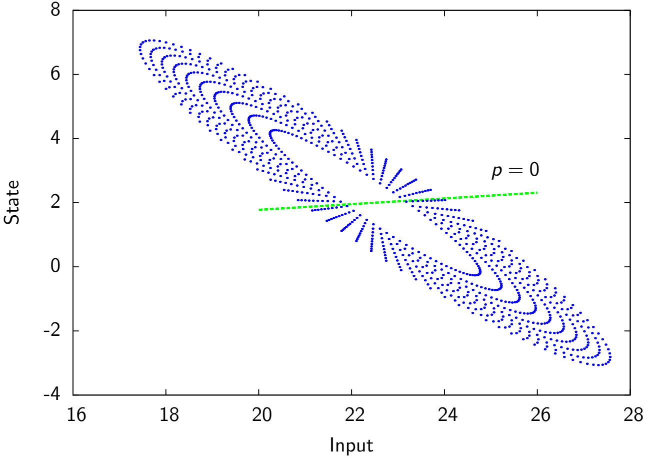
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State
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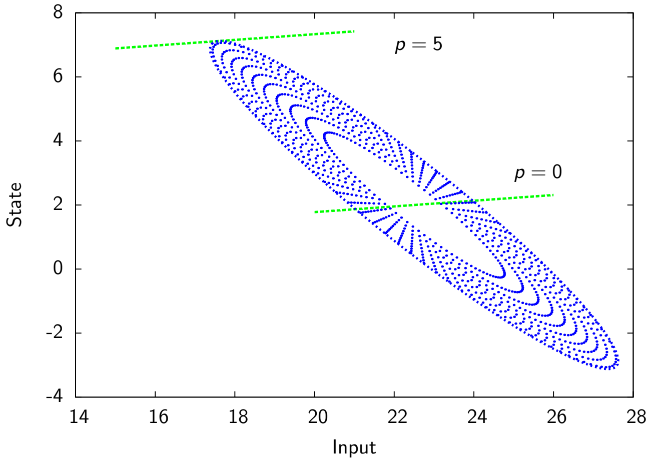
26
28
Input
$p = 0$


p = 5
p = 0
8
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-2
-4
State
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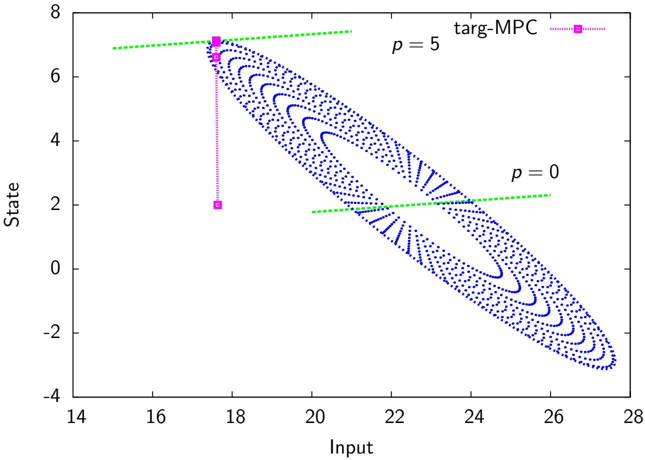
28
Input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State
Input
targ-MPC
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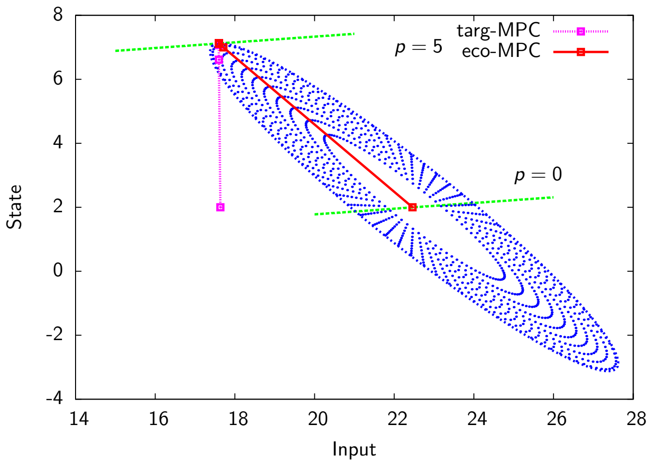

Performance Measures

	targ-MPC	eco-MPC	Δ(index)%
Loss[a]	$102.59	$48.722	52.5

[a] Reference: Maximum profit = 0

- Random disturbance corrupts state evolution
- All states assumed measured

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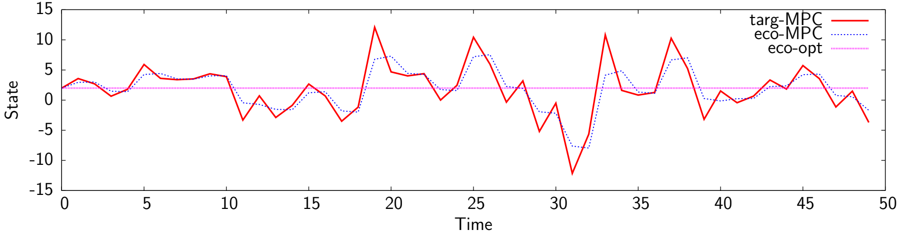
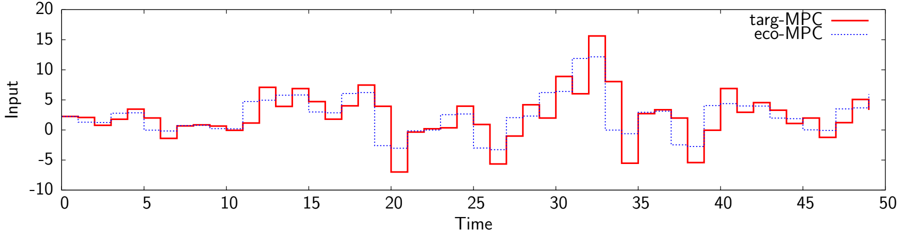
- All states assumed measured





	targ-MPC	eco-MPC	Δ(index)%
Loss[a]	\$2537.6	\$968.5	61.8

[a] Reference: Maximum profit = 0

Maximum throughput

- Consider a typical profit function for the plant:

$$(-L) = \sum_j p_{P_j} P_j - \sum_i p_{F_i} F_i - \sum_k p_{Q_k} Q_k$$

P_j: Product flows $\quad$ F_i: Feed flows $\quad$ Q_k: Utility duties

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P_j: Product flows F_i: Feed flows Q_k: Utility duties

- Assume all feed flows set in proportion to throughput (F), constant efficiency in the units and constant intensive variables

$$F_i = k_{F,i} F \qquad P_j = k_{P,j} F \qquad Q_k = k_{Q,k} F$$

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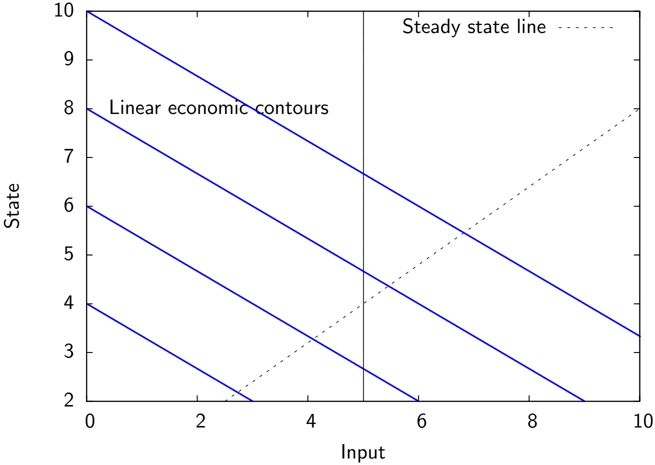
$$(-L) = \left(\sum_j p_{P_j} k_{P,j} - \sum_i p_{F_i} k_{F,i} - \sum_k p_{Q_k} k_{Q,k} \right) F = pF$$

p: operational profit per unit feed F processed

Economic optimum $\iff$ Maximizing throughput

- **Linear economics**: Unconstrained problem unbounded
- Constrained problem: Optimal solution lies on the process bounds



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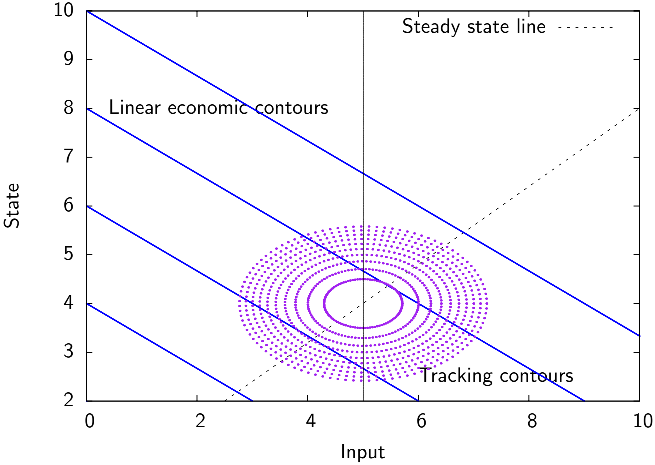

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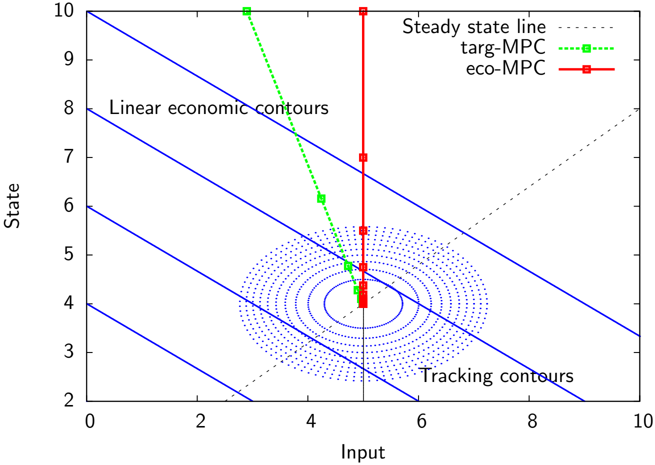


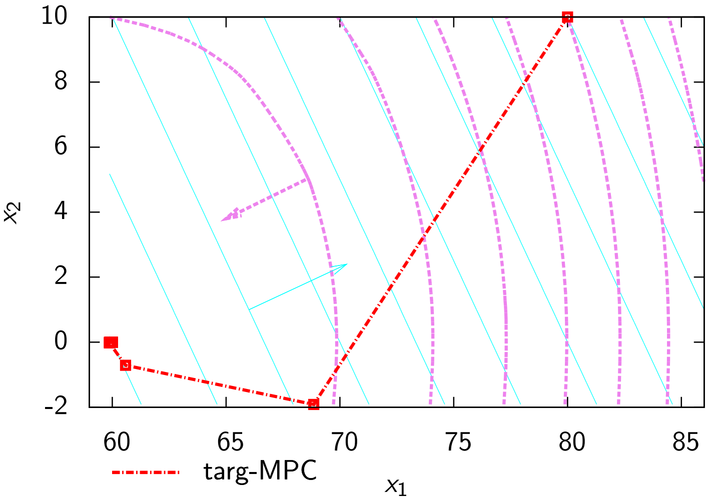
Example

$$\mathbf{x}_{k+1} = \begin{bmatrix} 0.857 & 0.884 \\ -0.0147 & -0.0151 \end{bmatrix} \mathbf{x}_k + \begin{bmatrix} 8.565 \\ 0.88418 \end{bmatrix} u_k$$

- Input constraint: $-1 \leq u \leq 1$

- $L_{eco} = \alpha' x + \beta' u$
- $\alpha = \begin{bmatrix} -3 & -2 \end{bmatrix}'$ $\quad \beta = -2$

- $L_{track} = \|x - x^*\|_Q^2 + \|u - u^*\|_R^2$
- $Q = 2I_2 \quad R = 2$
- $x^* = \begin{bmatrix} 60 & 0 \end{bmatrix}'$
- $u^* = 1$


10
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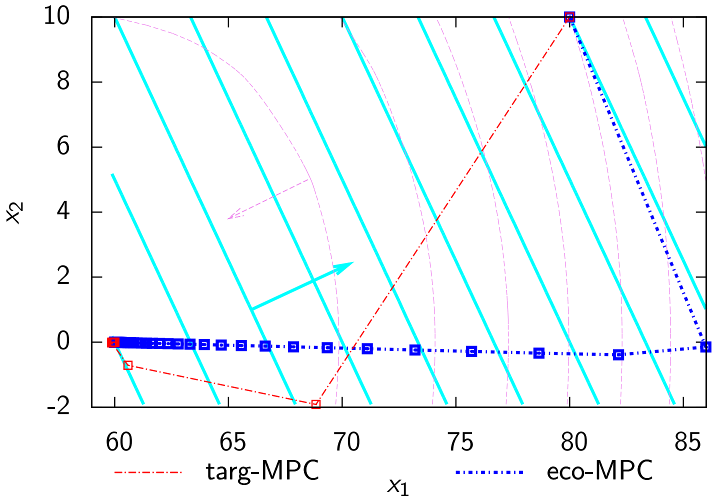
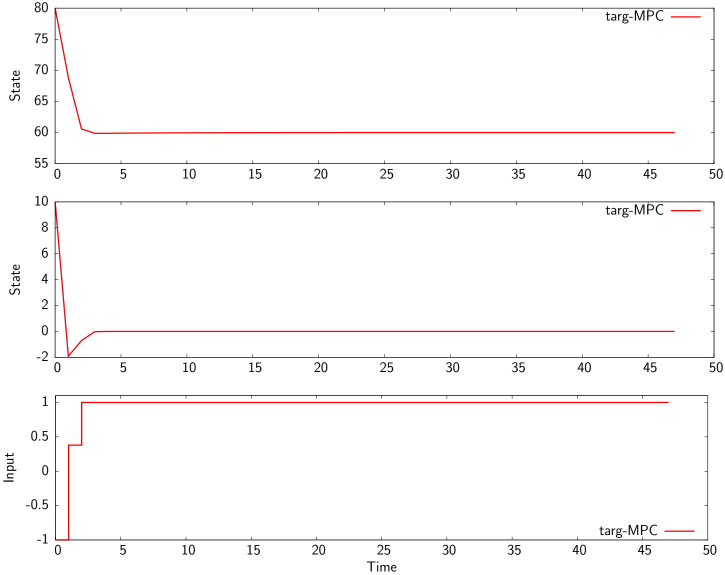
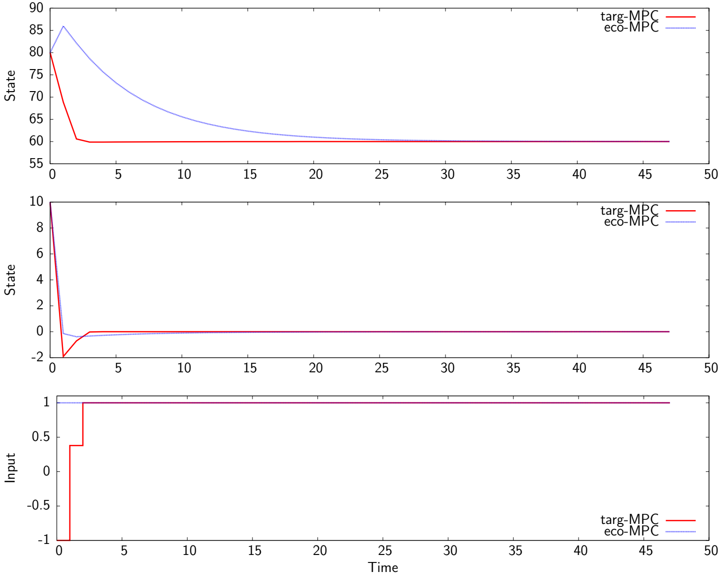
70
75
80
85
x_2
x_1
targ-MPC

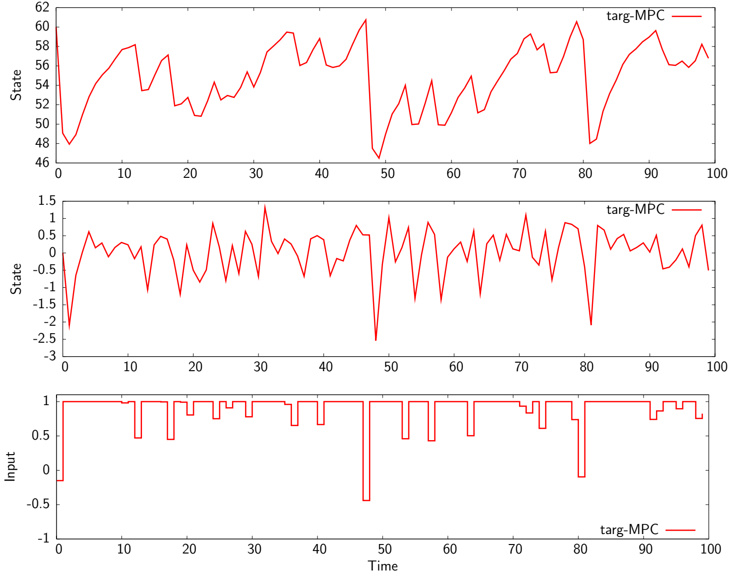

10
8
6
4
2
0
-2
x_2
60
65
70
75
80
85
x_1
targ-MPC
eco-MPC

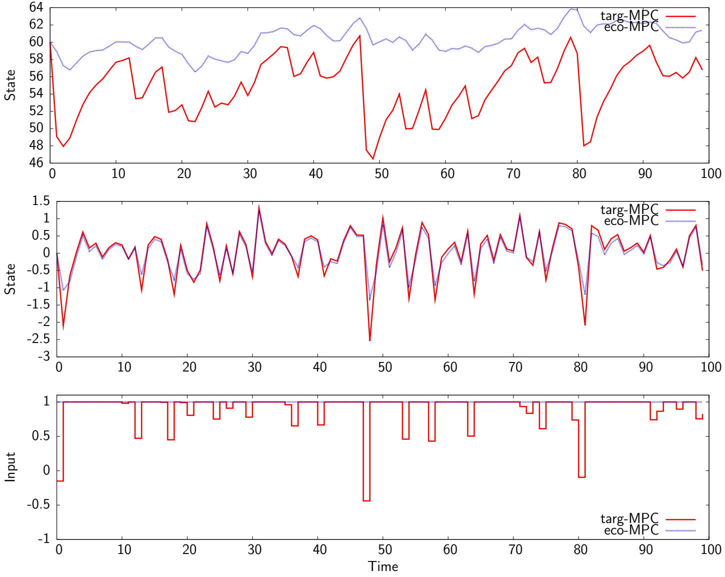

80
75
70
65
60
55
State
targ-MPC
0
5
10
15
20
25
30
35
40
45
50
10
8
6
4
2
0
-2
State
targ-MPC
0
5
10
15
20
25
30
35
40
45
50
1
0.5
0
-0.5
-1
Input
targ-MPC
0
5
10
15
20
25
30
35
40
45
50
Time


90
85
80
75
70
65
60
55
State
0 5 10 15 20 25 30 35 40 45 50
targ-MPC
eco-MPC
10
8
6
4
2
0
-2
State
0 5 10 15 20 25 30 35 40 45 50
targ-MPC
eco-MPC
1
0.5
0
-0.5
-1
Input
0 5 10 15 20 25 30 35 40 45 50
Time
targ-MPC
eco-MPC

Example: Effect of disturbance

- Random disturbance affecting the state evolution
- All states assumed measured
- System started at the steady optimum with zero disturbance


targ-MPC
State
Input
Time



State
64
62
60
58
56
54
52
50
48
46
0 10 20 30 40 50 60 70 80 90 100
targ-MPC
eco-MPC
State
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
-3
0 10 20 30 40 50 60 70 80 90 100
targ-MPC
eco-MPC
Input
1
0.5
0
-0.5
-1
0 10 20 30 40 50 60 70 80 90 100
Time
targ-MPC
eco-MPC

Future work

- Investigate economic models
  - Presented idea banks on a good economic measure
  - Translation of objectives needs deep investigation
  - Need to define a good representative of the process economics

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- Investigate economic models
  - Presented idea banks on a good economic measure
  - Translation of objectives needs deep investigation
  - Need to define a good representative of the process economics
- Establish asymptotic stability and convergence properties for broader class of cost functions
  - Steady state cost maybe nonzero $\Longrightarrow$ Infinite horizon cost is unbounded
  - Costs corresponding to the optimal input sequence may not be monotonically decreasing

- Update the software tools to handle the new class of problems
  - Efficient software tools critical to the evaluation of the new class of problems
  - The existing tools handle quadratic objective functions
  - Economics may not be quadratic and hence the tools have to be capable of handling more general cost functions

- Set up the problem for a realistic scenario and test using industrial data
  - Simulations, like the ones shown, just predict the possible advantages of the new scheme
  - The idea must be tested for a physical system with well defined economics
- Collaborate for the distributed version
  - Distributed control schemes allow more robust and flexible control
  - The new scheme can be implemented in distributed scenario

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- A separate layer causes a loss in economic performance during transient
- Opportunity to rethink distribution of functionality between layers
- Merging the economics with the controller objective reduces the loss of economic information
- Economic optimizing control expected to capture the potential profitable areas of operation