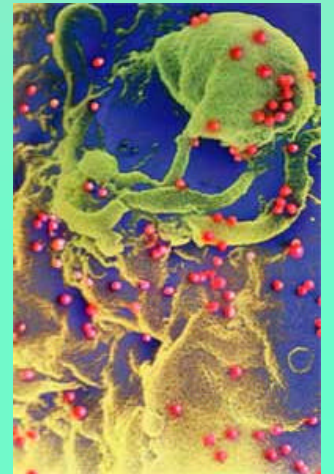


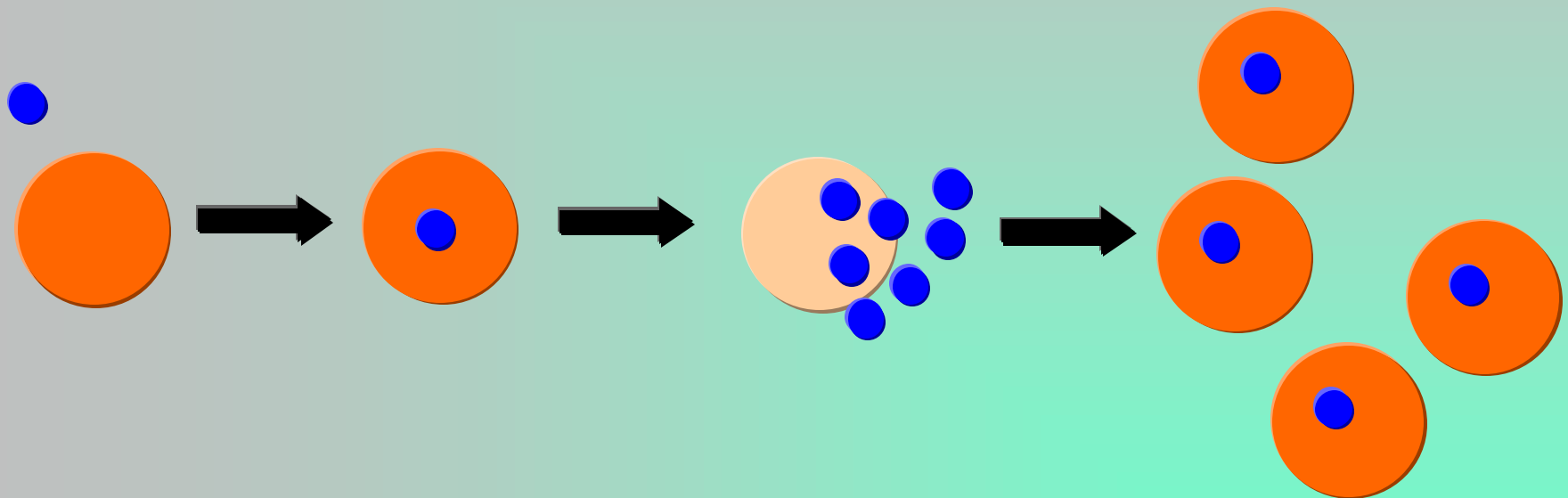
# Dynamics of HIV infection

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Martin Nowak  
Institute for Advanced Study  
Princeton

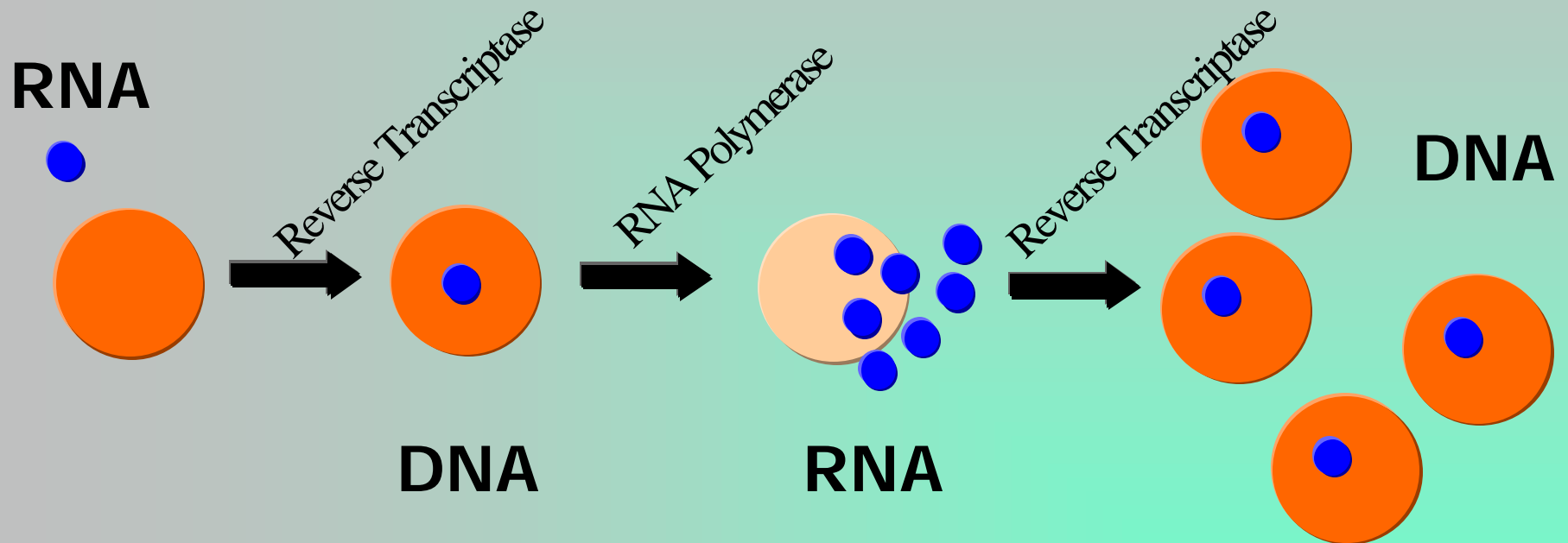


# The basic idea of a virus:



'Replicate me!'

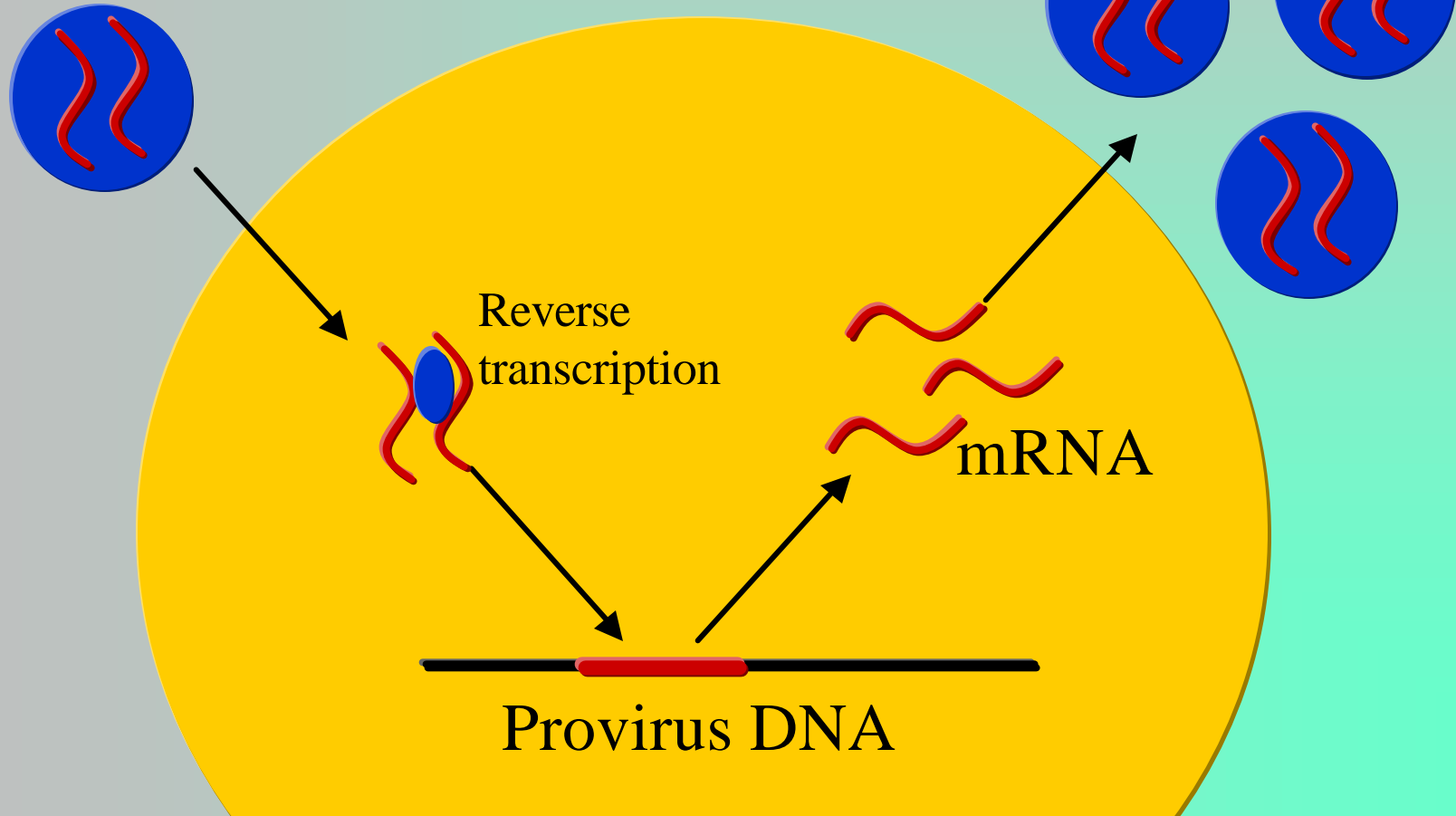
# The basic idea of a retrovirus:



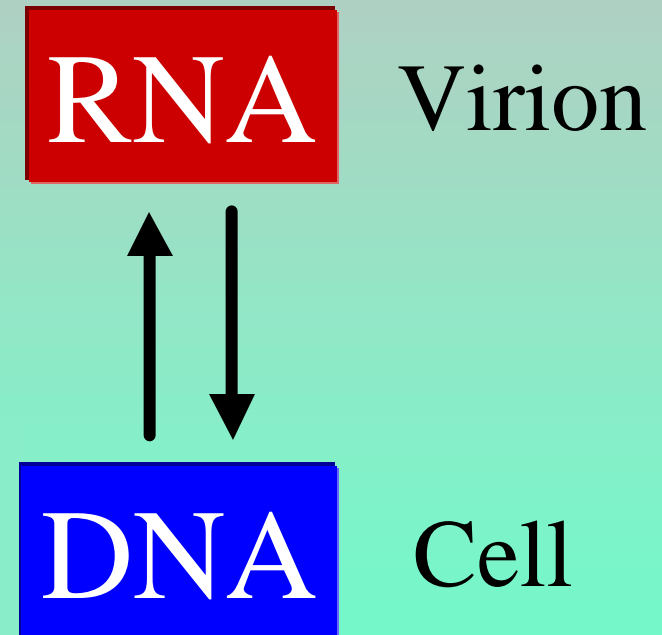
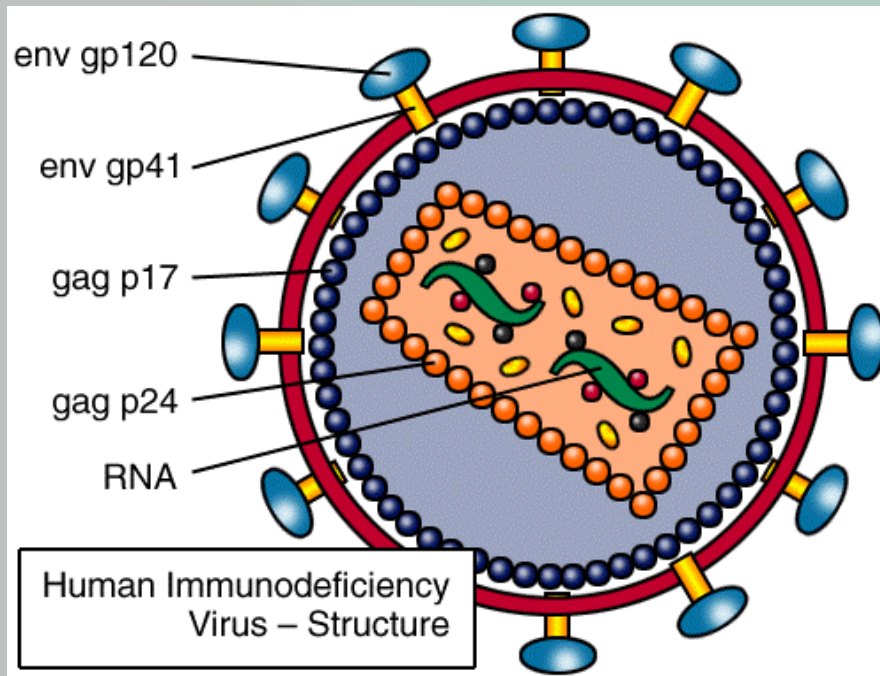
'Reverse transcribe me!  
Integrate me!  
RNA polymerase me!'

# The basic idea of a retrovirus:

Virus RNA



# HIV is a retrovirus



Viral DNA integrates into the host cell genome.

# Images of HIV



# Human immunodeficiency virus (HIV)



- ⌘ HIV was discovered in 1984.
- ⌘ Now there are 35 million infected, 15 million dead.
- ⌘ There is anti-viral therapy with problematic side effects.
- ⌘ There is no vaccine.

# Retroviruses



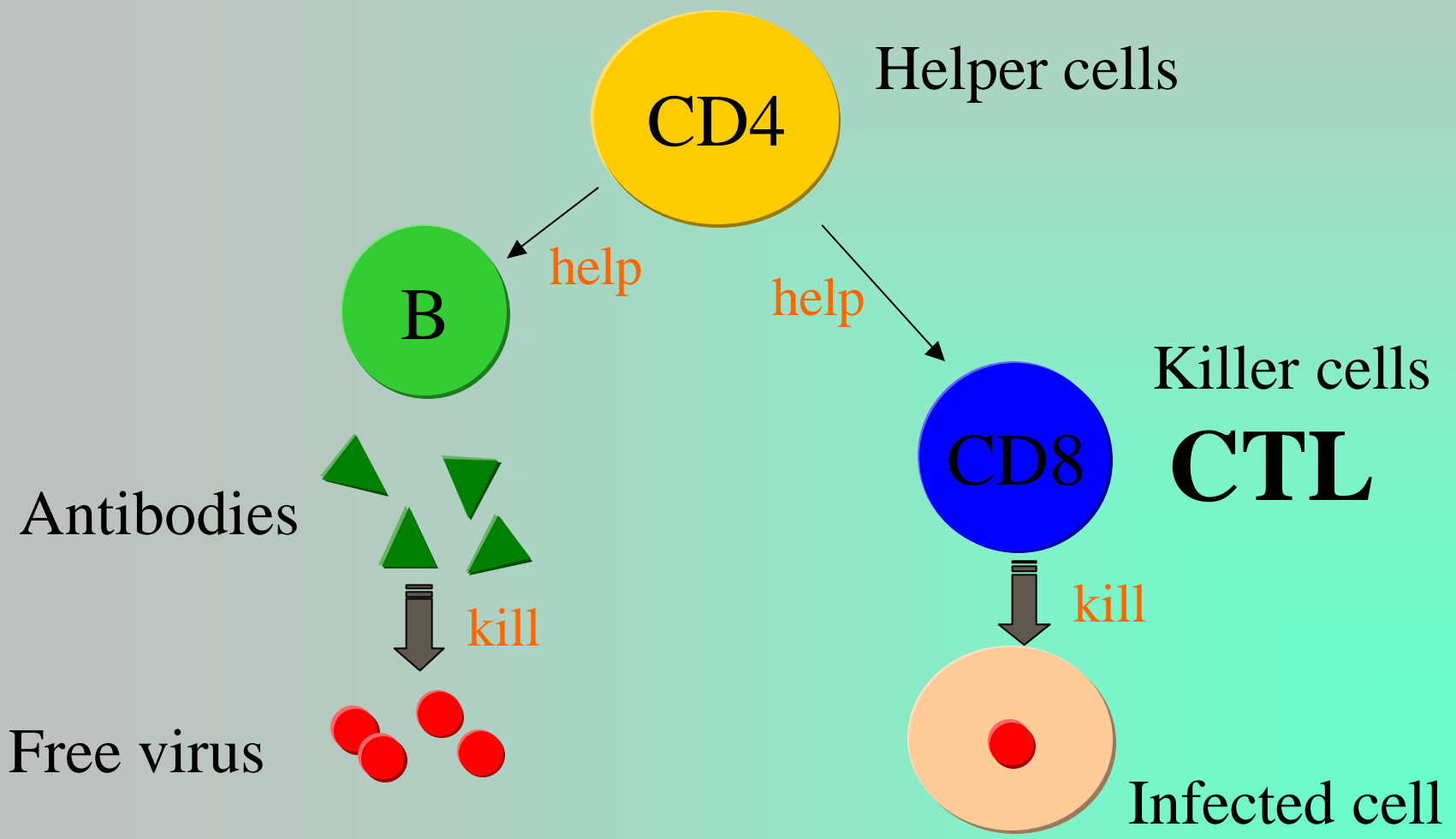
⌘ Oncoviruses (HTLV)

⌘ Lentiviruses (HIV, SIV, FIV, EIAV, ...)

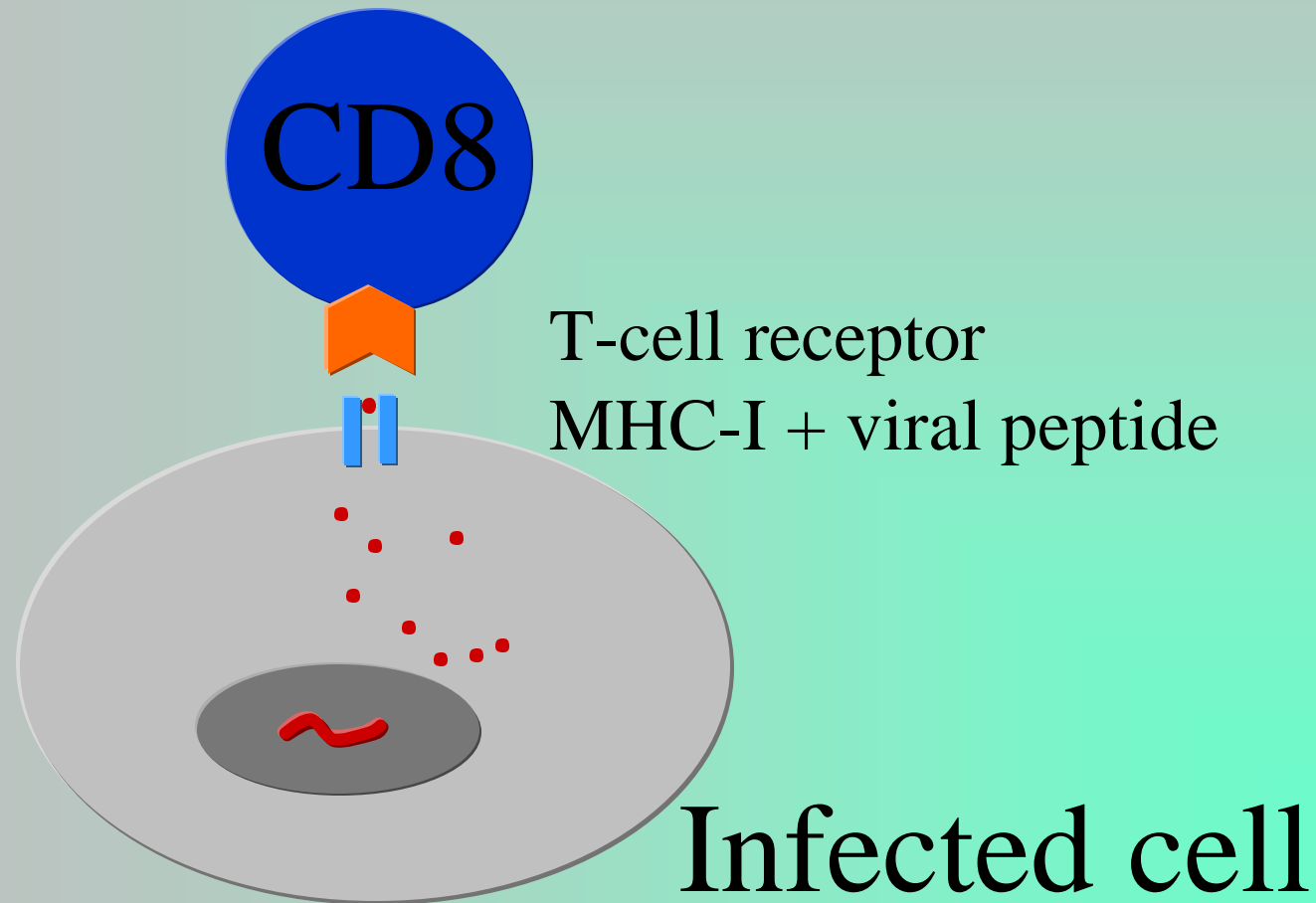
⌘ Spumaviruses



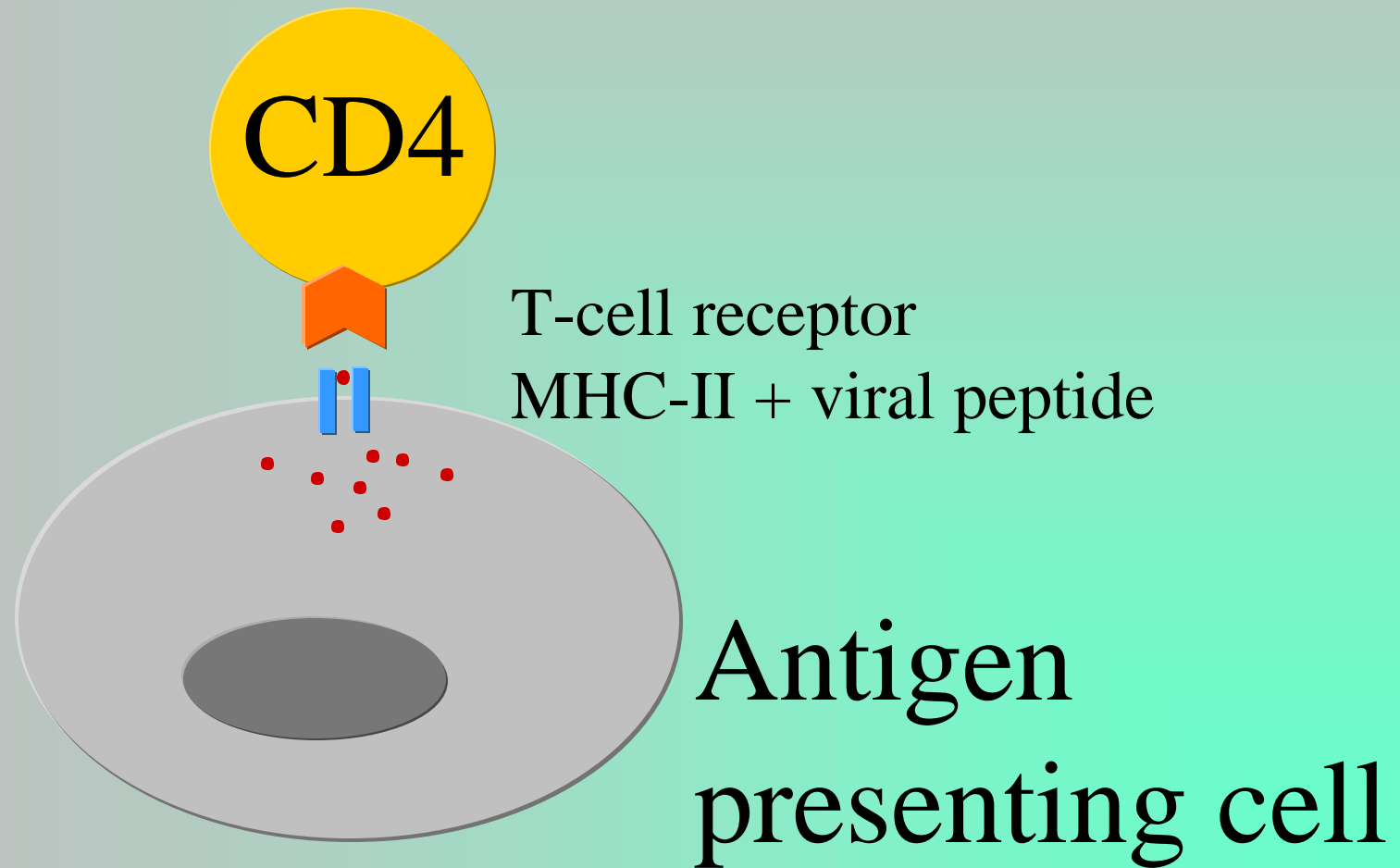
# Viruses are opposed by immune responses



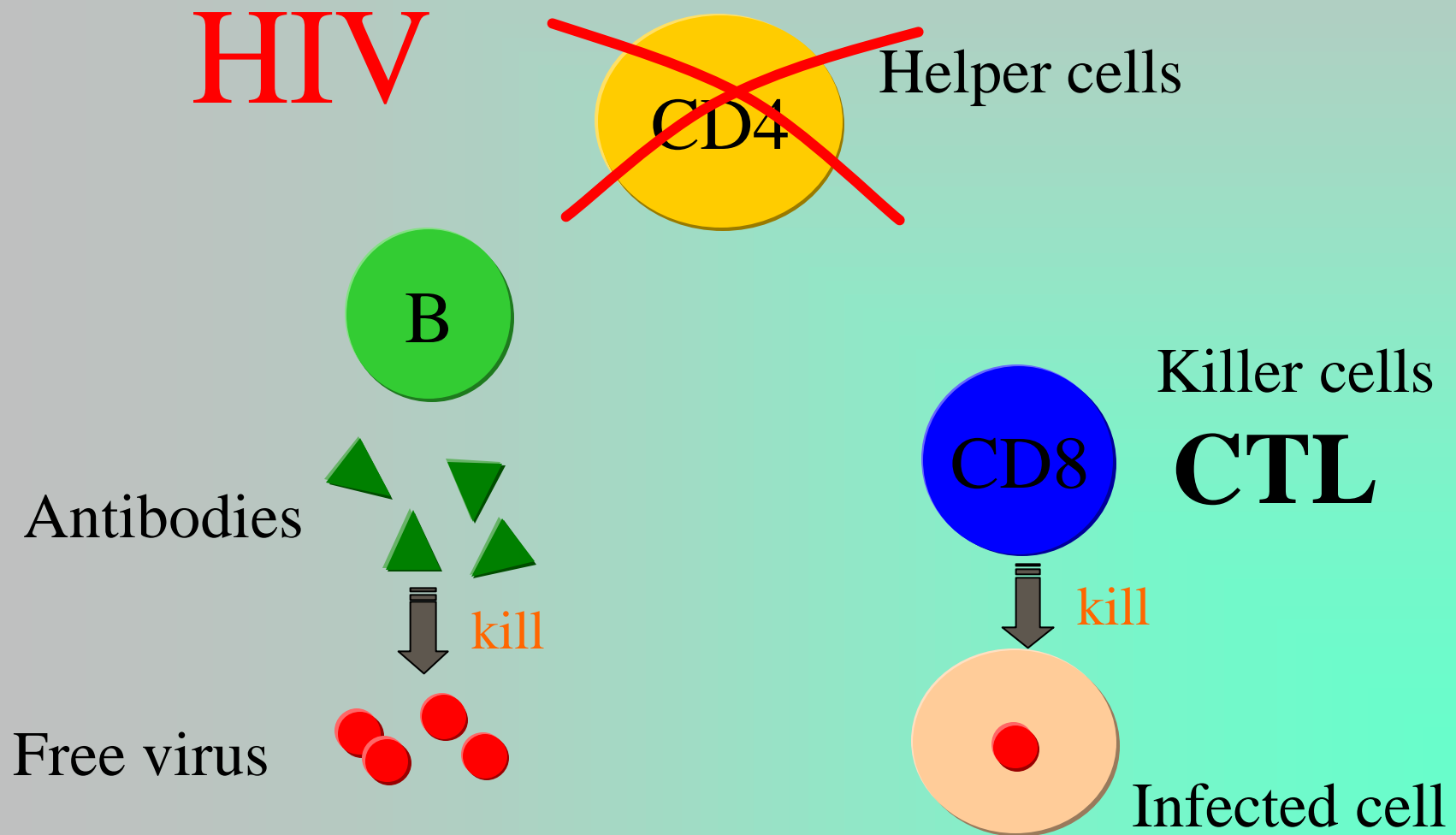
# CD8 cell recognition



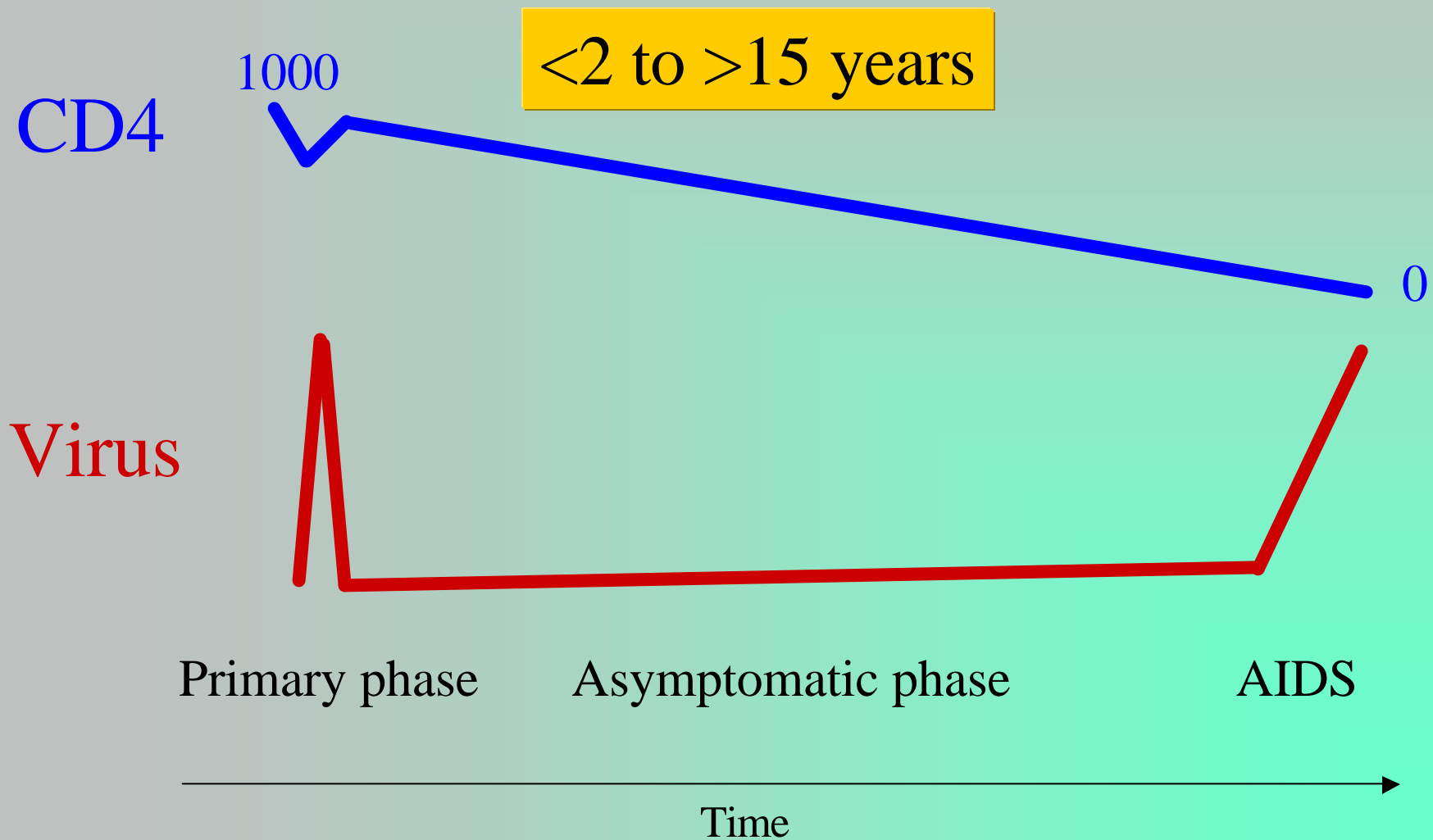
# CD4 cell recognition



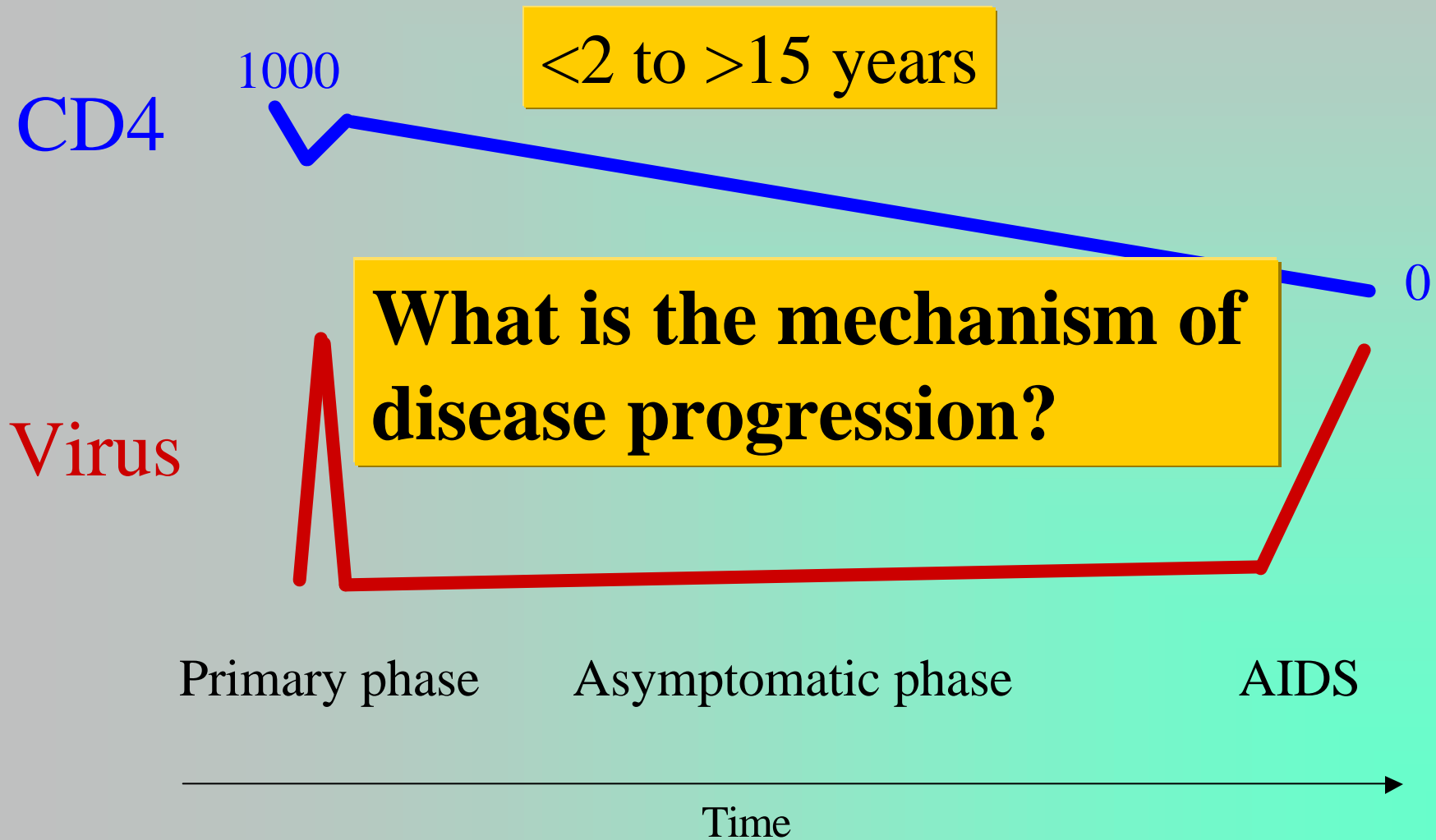
# HIV kills CD4 cells



# HIV-1: clinical profile



# HIV-1: clinical profile



# HIV-1: disease progression



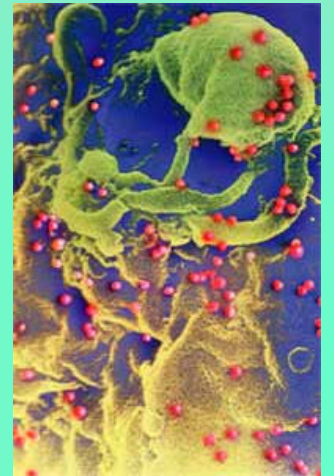
**Fast progressors: high virus load**

What makes the difference?  
Can treatment help?

**Slow progressors: low virus load**

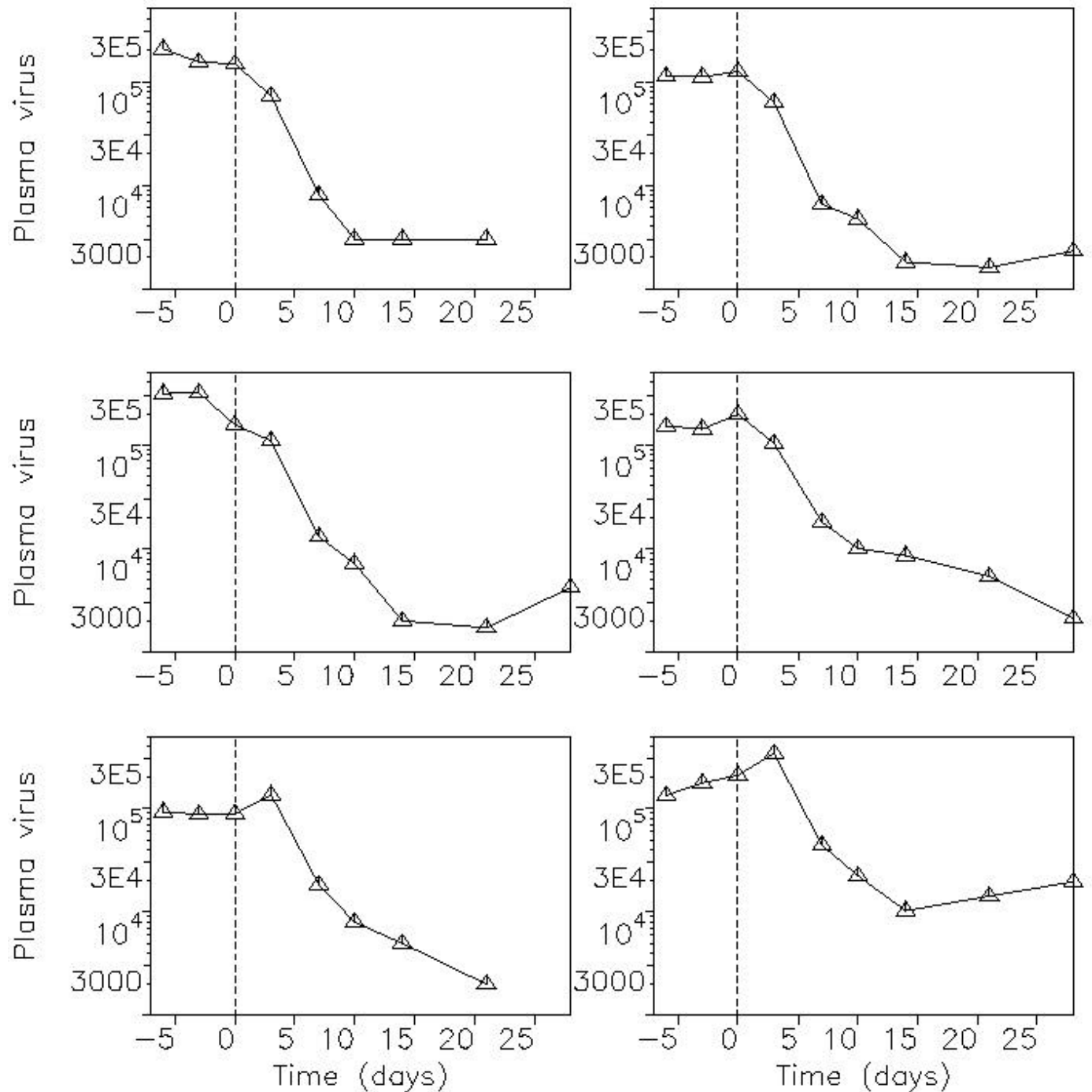
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How fast does HIV-1  
reproduce in vivo ?





1994:  
protease  
inhibitors  
and  
quantitative  
PCR



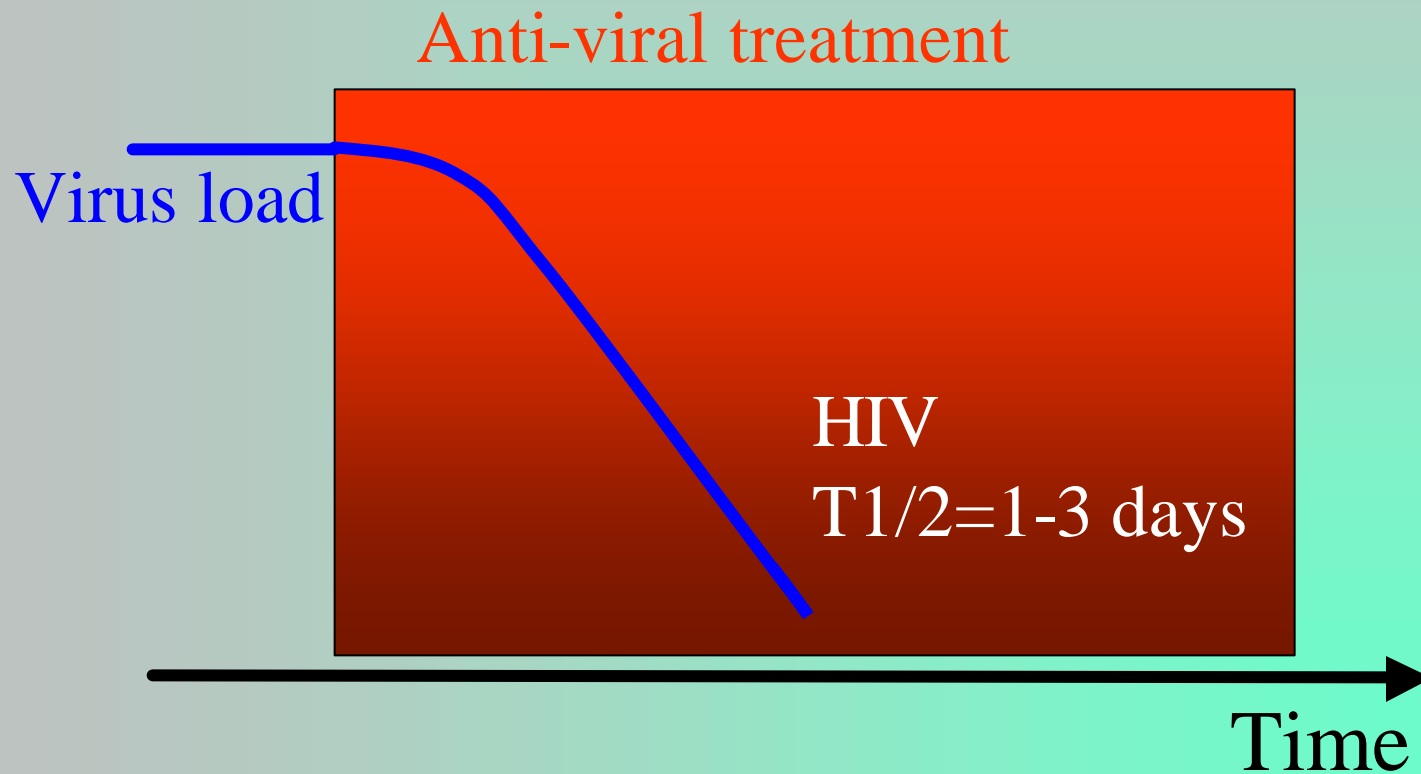
George Shaw

# Anti-HIV drugs

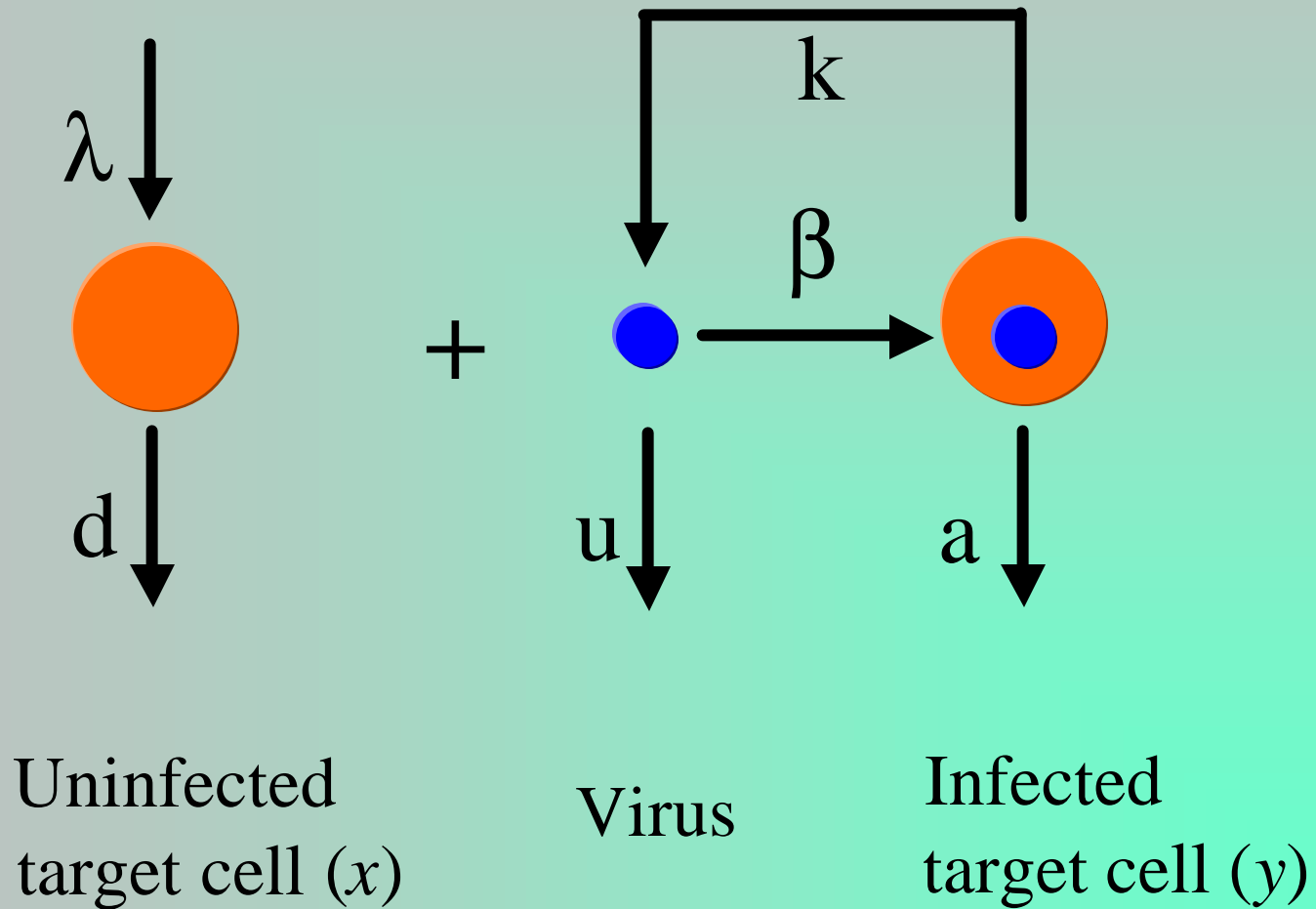


- ⌘ Reverse transcriptase inhibitors
- ⌘ Protease inhibitors

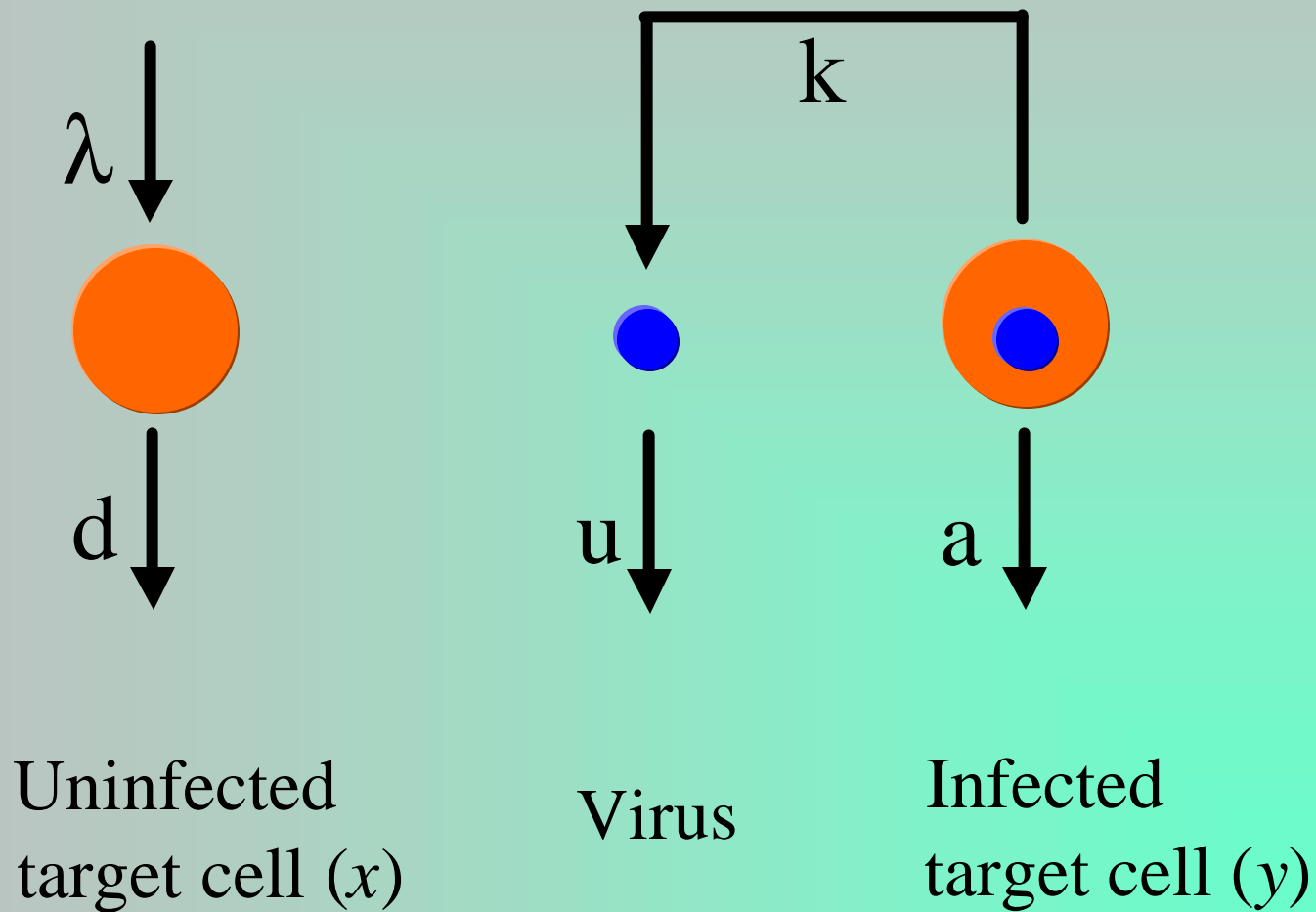
# Treatment leads to a rapid decline in virus load



# Virus dynamics



# Virus dynamics with treatment



# The basic model of virus dynamics

Uninfected cells  $\dot{x} = l - dx - b xv$

Infected cells  $\dot{y} = b xv - ay$

Free virus  $\dot{v} = ky - uv$

**Micro-epidemiology  
within infected host**

# Anti-viral treatment

Uninfected cells  $\dot{x} = l - dx - \cancel{bxv}$

Infected cells  $\dot{y} = \cancel{bxv} - ay$

Free virus  $\dot{v} = ky - uv$

# Virus decline

Infected cells  $\dot{y} = -ay$

Free virus  $\dot{v} = ky - uv$

**Analytic solution**

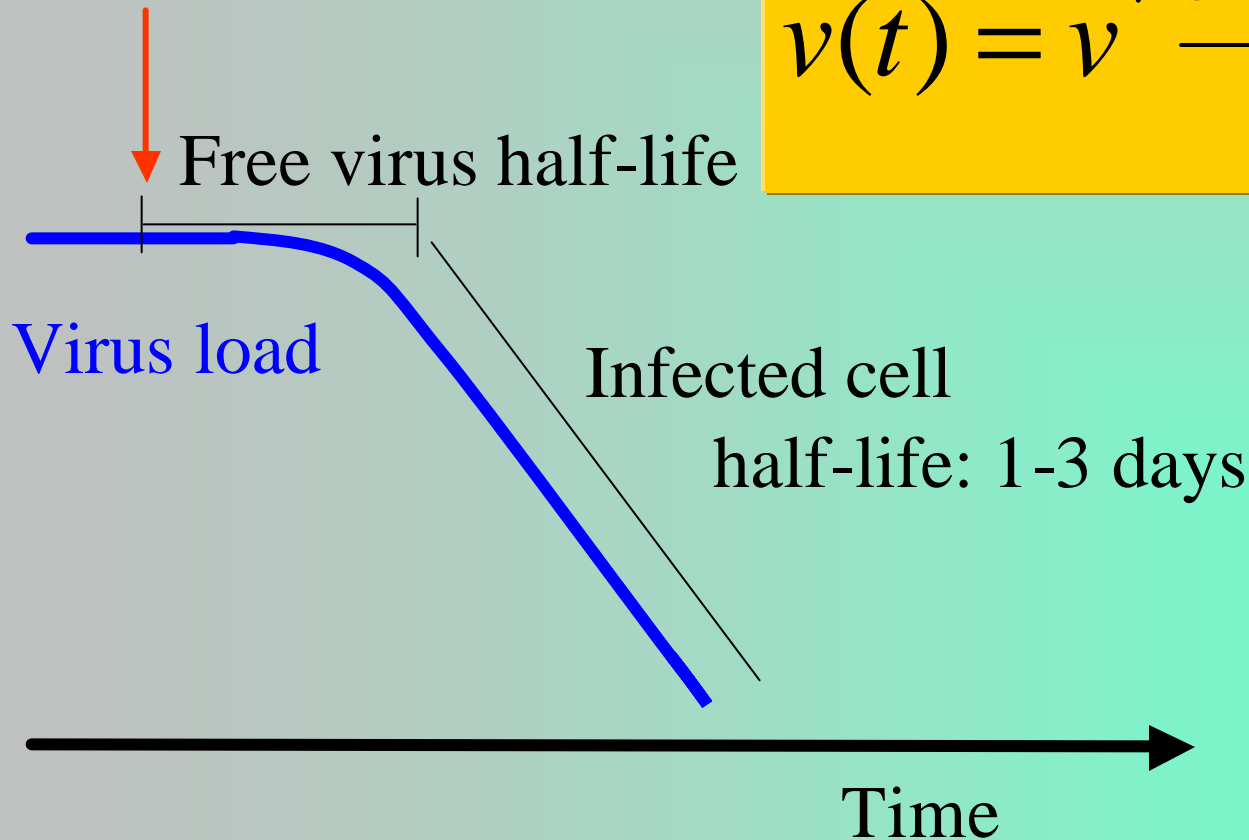
$$y(t) = y(0)e^{-at}$$

$$v(t) = v(0) \frac{ue^{-at} - ae^{-ut}}{u - a}$$



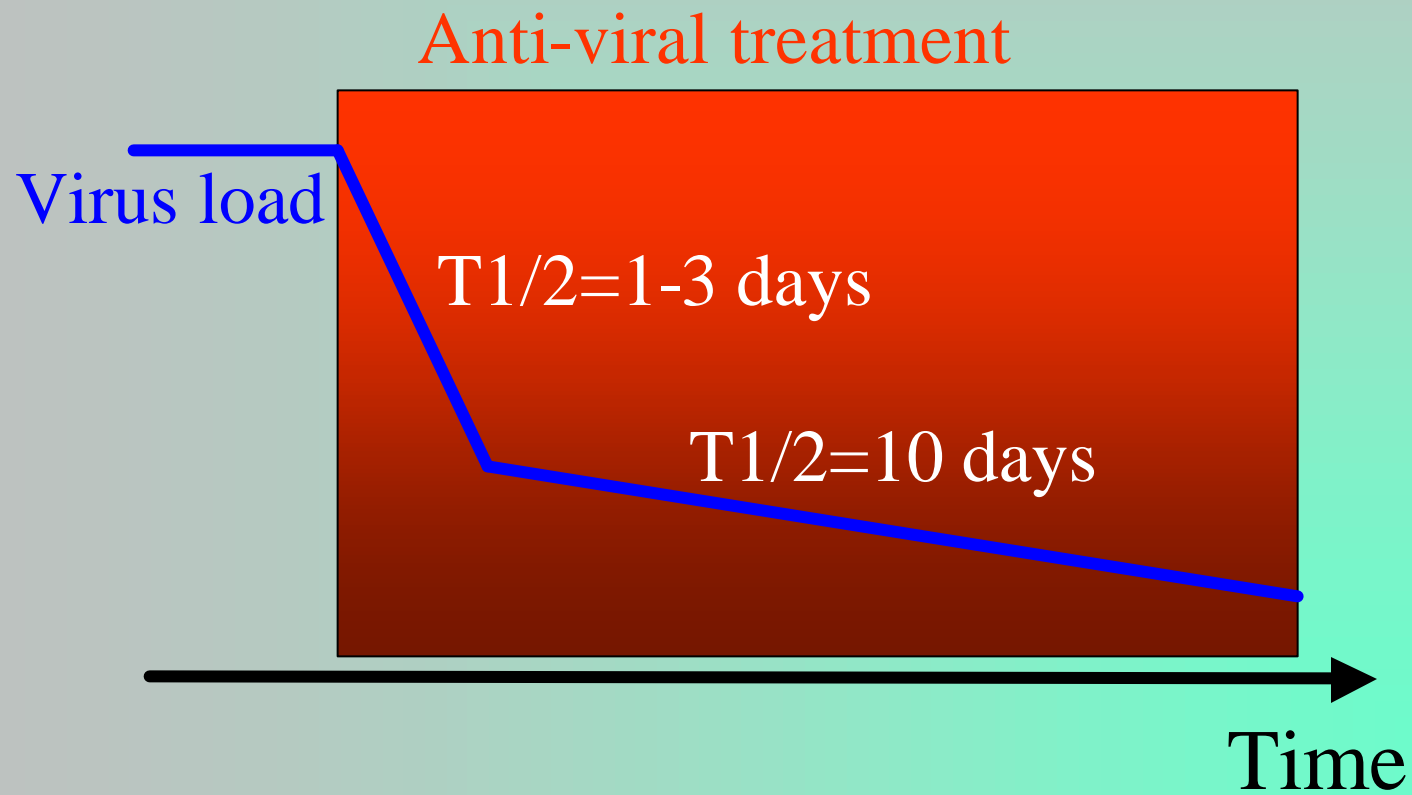
# Virus declines as

Anti-viral treatment



$$v(t) = v^* \frac{ue^{-at} - ae^{-ut}}{u - a}$$

# Latently infected cells



# An extended model of virus dynamics

Uninfected cells

$$\dot{x} = \lambda - dx - \mathbf{b} xv$$

Productively infected cells

$$\dot{y}_1 = q_1 \mathbf{b} xv - a_1 y_1 + \mathbf{a} y_2$$

Latently infected cell

$$\dot{y}_2 = q_2 \mathbf{b} xv - a_2 y_2 - \mathbf{a} y_2$$

Cells with defective provirus

$$\dot{y}_3 = q_3 \mathbf{b} xv - a_3 y_3$$

Free virus

$$\dot{v} = ky_1 - uv$$

# HIV-1 half-lives



- ⌘ Productively infected cells : 1-3 days
- ⌘ Latently infected cells : 10 days
- ⌘ Defective provirus : 100 days
- ⌘ Free virus : hours

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HIV eradication requires 1-3 years of effective therapy.

# HIV-1 half-lives

- ⌘ Productively infected cells : 1-3 days
- ⌘ Latently infected cells : 10-100 days
- ⌘ Defective provirus : 100 days
- ⌘ Free virus : hours

HIV eradication requires >10 years of effective therapy and is most likely impossible.

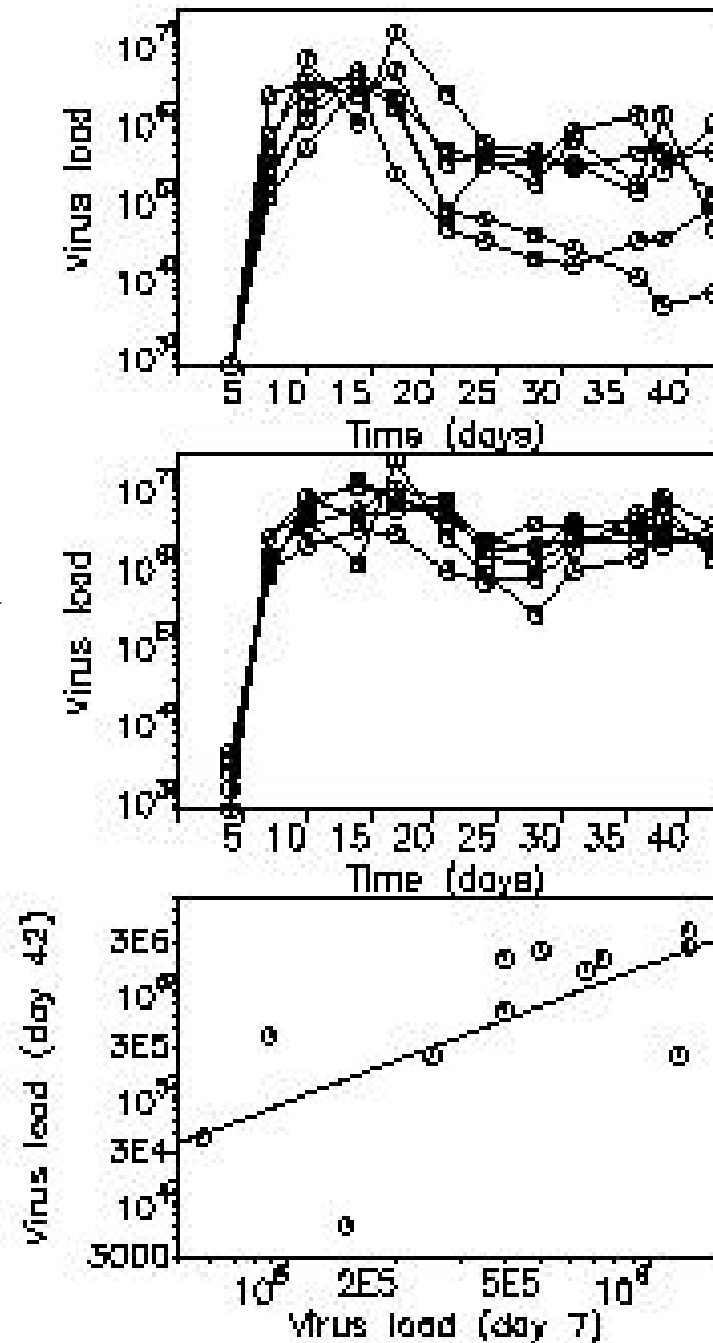
# SIV

simian immunodeficiency virus

Study of primary infection shows correlation between virus load in 1st week of infection and virus load at set point after the initial peak.

Virus load at set point is strongly correlated with survival time.

Hence survival time can be predicted from virus growth rate in the first week of infection.



# Use basic model to study primary infection

**Uninfected cells**

$$\dot{x} = l - dx - b xv$$

**Infected cells**

$$\dot{y} = b xv - ay$$

**Free virus**

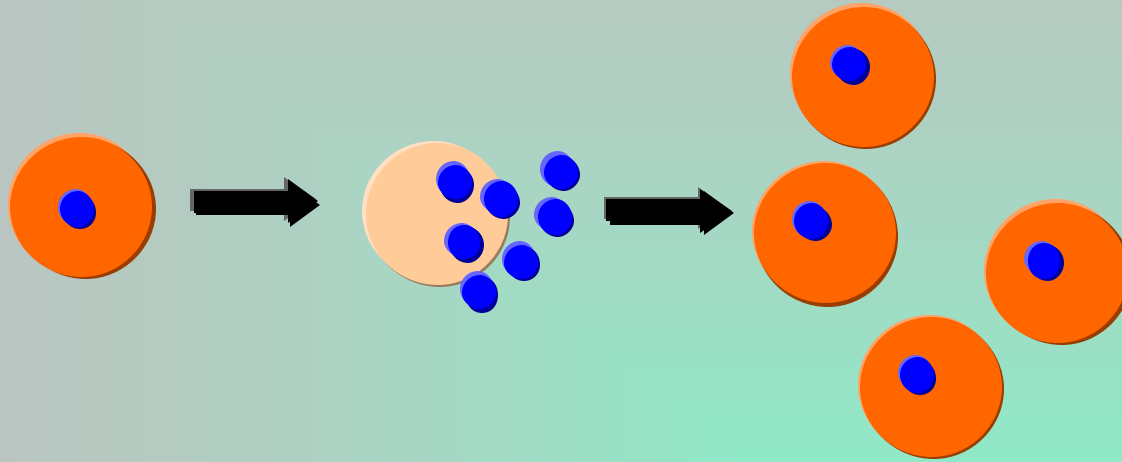
$$\dot{v} = ky - uv$$

**Initial conditions**

$$x(0) = l / d \quad y(0) = 0 \quad v(0) = 0$$



# Basic reproductive rate (or ratio or number)

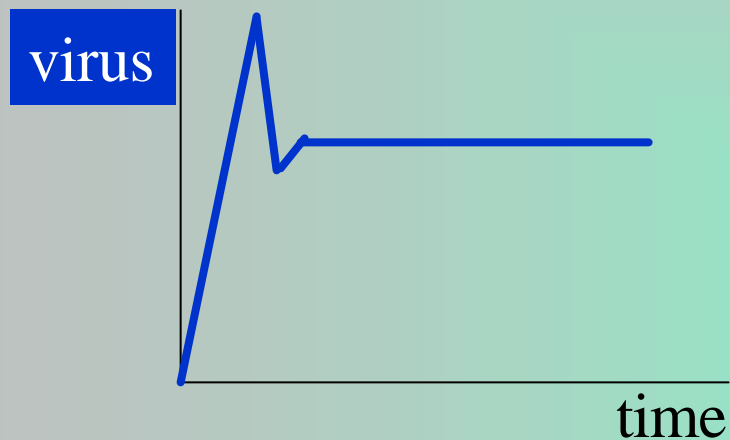


= the number of newly infected cells that arise from one infected cell if most cells are uninfected

$$R_0 = \frac{bk}{au} x(0) = \frac{bkl}{aud}$$

# Basic reproductive rate

Infection takes place if  $R_0 > 1$



The system goes in damped oscillations to the equilibrium:

$$x^* = \frac{x(0)}{R_0}, \quad y^* = (R_0 - 1) \frac{du}{bk}, \quad v^* = (R_0 - 1) \frac{d}{b}$$