Dynamical Systems with Symmetry - ME225DS
Winter 2008

Homework #2 - Due Thursday, January 24, in class

1. (5 pts) Prove that $\{2^m 3^n : m, n \in \mathbb{Z}\}$ is a group under multiplication.

2. (a) (10 pts) Fill in the multiplication table (see next page) for the dihedral group

$$\mathrm{D}_4 = \langle \gamma_1, \gamma_2 \rangle = \{e, \gamma_2, \gamma_2^2, \gamma_2^3, \gamma_1, \gamma_1\gamma_2, \gamma_1\gamma_2^2, \gamma_1\gamma_2^3\}$$

with

$$\gamma_1^2 = e, \qquad \gamma_2^4 = e, \qquad \gamma_2\gamma_1\gamma_2 = \gamma_1.$$

Each entry should be a member of the set $\{e, \gamma_2, \gamma_2^2, \gamma_2^3, \gamma_1, \gamma_1\gamma_2, \gamma_1\gamma_2^2, \gamma_1\gamma_2^3\}$.

(b) (10 pts) Fill in the multiplication table (see next page) for the quaternion group

$$Q = \{1, i, j, k, -1, -i, -j, -k\}$$

with

$$i^2 = j^2 = k^2 = ijk = -1.$$

Here, following convention, the identity element is 1. Furthermore, the $-$ sign can always be pulled to the left side, for example $j(-k) = -jk$. Each entry should be a member of the set $\{1, i, j, k, -1, -i, -j, -k\}$.

(c) (5 pts) Are D_4 and Q isomorphic to each other? Please explain your reasoning.

3. (20 pts) Find all subgroups of the dihedral group $\mathrm{D}_6 = \langle \gamma_1, \gamma_2 \rangle$ with

$$\gamma_1^2 = e, \qquad \gamma_2^6 = e, \qquad \gamma_2\gamma_1\gamma_2 = \gamma_1.$$

Please list the subgroups in terms of the elements

$$\{e, \gamma_2, \gamma_2^2, \gamma_2^3, \gamma_2^4, \gamma_2^5, \gamma_1, \gamma_1\gamma_2, \gamma_1\gamma_2^2, \gamma_1\gamma_2^3, \gamma_1\gamma_2^4, \gamma_1\gamma_2^5\}.$$

4. (a) (5 pts) Is the intersection of a set of subgroups of a group also a subgroup of the same group? If so, prove this. If not, provide a counterexample.

(b) (5 pts) Is the union of a set of subgroups of a group also a subgroup of the same group? If so, prove this. If not, provide a counterexample.

	e	γ_2	γ_2^2	γ_2^3	γ_1	$\gamma_1\gamma_2$	$\gamma_1\gamma_2^2$	$\gamma_1\gamma_2^3$
e								
γ_2								
γ_2^2								
γ_2^3								
γ_1								
$\gamma_1\gamma_2$								
$\gamma_1\gamma_2^2$								
$\gamma_1\gamma_2^3$								

	1	i	j	k	-1	$-i$	$-j$	$-k$
1								
i								
j								
k								
-1								
$-i$								
$-j$								
$-k$								