Dynamical Systems with Symmetry - ME225DS
Winter 2008

Homework #3 - Due Thursday, January 31, in class

1. Consider the symmetric group S_8.

(a) (5 pts) Simplify the product

$$\pi_1 = (1,2,3)(1,2,8)(3,7)(2,7)(1,2,4,5,8)(7,8)(1,3,2)(1,7)$$

by writing it as a product of disjoint cycles.

(b) (5 pts) Find the inverse of the element

$$\pi_2 = (1,3,4)(2,7).$$

2. Consider the group

$$\mathrm{D}_4 = \langle \gamma_1, \gamma_2 \rangle = \{e, \gamma_2, \gamma_2^2, \gamma_2^3, \gamma_1, \gamma_1\gamma_2, \gamma_1\gamma_2^2, \gamma_1\gamma_2^3\}$$

with

$$\gamma_1^2 = e, \qquad \gamma_2^4 = e, \qquad \gamma_2\gamma_1\gamma_2 = \gamma_1.$$

(a) (10 pts) Find all normal subgroups of D_4, and for each normal subgroup H find the quotient group D_4/H.

(b) (5 pts) Let H be a subgroup of Γ. The normalizer $N(H)$ of H is

$$N(H) = \{\gamma \in \Gamma : \gamma^{-1}H\gamma = H\}.$$

(Note that, in general, the normalizer is the largest subgroup of Γ that has H as a normal subgroup.) For $\Gamma = \mathrm{D}_4$, what is the normalizer of $H = \{e, \gamma_1\}$?

(c) (5 pts) What is the quotient group $N(\{e, \gamma_1\})/\{e, \gamma_1\}$?

3. (10 pts) Let H be a subgroup of Γ, and let $\gamma_0 \in \Gamma$. The set

$$H\gamma_0 = \{h\gamma_0 : h \in H\}$$

is called the *right coset* of H determined by γ_0. Prove that two right cosets $H\gamma_1$ and $H\gamma_2$ are either identical or have no elements in common.

4. (20 pts) Describe all groups of order 21.