Fundamentals study guide for midterm

Consider the following fundamental equations:

$$ndU = \delta Q + \delta W \qquad n\Delta U = Q + W$$

$$W = -\int P_{ext} dV^t \qquad H = U + PV$$

$$C_V \equiv \left(\frac{dU}{dT}\right)_V \qquad C_P \equiv \left(\frac{dH}{dT}\right)_P$$

Consider also the specific case of an ideal gas, whose fundamental relations are:

$$PV = RT \qquad U = U(T) \text{ only}$$

In the following problems, use only these fundamentals as methods to reach your final answer. Avoid your book and notes; rather, challenge yourself to get to the answer independently.

Problem I

An ideal gas with constant heat capacity starts out at T_1 and P_1 (and $V_1 = RT_1/P_1$). Derive for each of the following the correct expressions for the following seven quantities: $T_2, P_2, V_2, \Delta U$, ΔH, W/n, Q/n.

(a) for a quasi-static isochoric process, if T_2 is given.

(b) for a quasi-static isobaric process, if T_2 is given.

(c) for a reversible isothermal process, if P_2 is given.

(d) for a reversible adiabatic process, if T_2 is given.

Problem 2

Prove the following for an ideal gas: (a) $H(T)$ only, and (b) $C_P = C_V + R$.

Problem 3

Show that $Q = n\Delta U$ and $Q = n\Delta H$ for quasi-static constant-volume and constant=pressure heating processes, respectively.

Problem 4

Heat per mole in the amount of Q/n is added to a system held at volume V. The system changes from $T_1, P_1 \rightarrow T_2, P_2$. Find ΔU and ΔH if the system is (a) a nonideal gas and (b) an incompressible liquid.

Problem 5

An ideal gas is inside a piston maintained at constant temperature. If a compression process occurs but is not quasi-static (e.g., very fast), can one use the equation $W/n = RT\ln(V_1/V_2)$ to compute the work? Why or why not? Think of an example that demonstrates your reasoning.

Problem 6

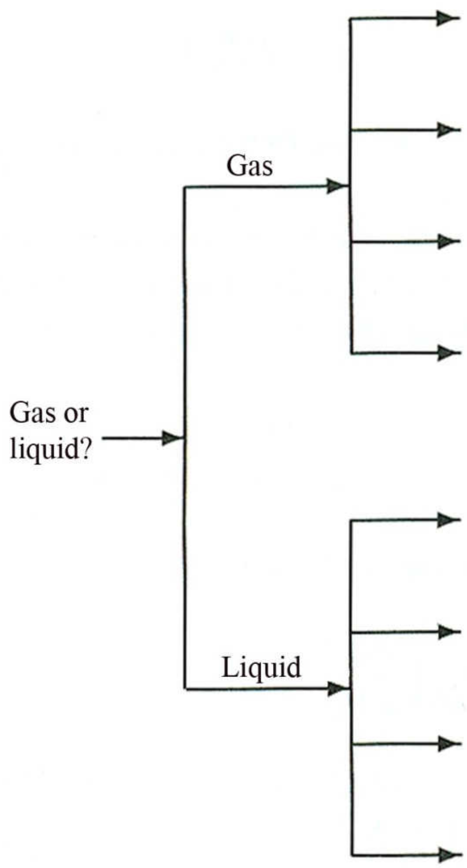
An ideal gas undergoes the following quasi-static three-step cycle: (i) adiabatic expansion, (ii) isobaric expansion, (iii) isothermal compression. Draw the process on a PV diagram and write an expression for the net work in terms of the pressures and temperatures at each point. Is the net work positive or negative?

Problem 7

An ideal gas monatomic gas enters a turbine with velocity u_1 through a opening of radius R_1. Its entering conditions are T_1 and P_1. The gas exits the turbine through an opening of radius R_2 and at T_2, P_2. Derive an expression for the work the turbine can generate per mole of gas.

Problem 8 (SVA 3.64)

Shown below is the *Equation-of-State Decision Tree.* For each item, indicate when each should be used and their relative advantages/disadvantages in process calculations.



(a) Ideal gas

(b) 2-term virial equation

(c) Cubic equation of state

(d) Lee/Kesler tables, Appendix E

(e) Incompressible liquid

(f) Rackett equation, Eq. (3.72)

(g) Constant β and κ

(h) Lydersen et al. chart, Fig. 3.16

Problem 9

An ideal monatomic gas with molecular weight M_w flows steadily through a well-insulated pipe of constant diameter. At point 1, the gas has velocity u_1 and molar volume V_1. Due to frictional losses, the velocity at point 2 is reduced by 10%. Assume constant heat capacities.

(a) Write an expression for the temperature increase $T_2 - T_1$.

(b) Write an expression for the pressure change P_2/P_1 in terms of T_1 and T_2.

Problem 10

At constant pressure, an ideal microwave delivers $500\ W$ of energy to an $8\ oz$ glass of water ($n \approx 13\ mol$) initially at $20\ ℃$. At atmospheric pressure, the latent heat of vaporization of water is $\Delta H_{\text{vap}} = 40.6\ kJ/mol$. How long will it take for the entire glass to vaporize if the liquid has a constant heat capacity $C_P = 75\ J/mol\ K$? Show a full analysis beginning with the first law.